Mathematics 3200H – Number Theory
TRENT UNIVERSITY, FALL 2022

Assignments

The references below are to the textbook: *Elementary Number Theory* (2nd Edition), by Underwood Dudley. Please show all your work. Unless stated otherwise on a given assignment, you are permitted to work together and look things up, so long as you write up your solutions by yourself and acknowledge all sources and help that you ended up using. You may submit your solutions on paper or via Blackboard, or – as a last resort! – by email to the instruictor at `sbilaniuk@trentu.ca` .

For each n with $1 \leq n \leq 11$, Assignment #n will normally be due at the end of week $n+1$ of the course.

Assignment #1. *Due on Friday, 16 September.*

§1 Exercise # 2; Problems # 3, 14, 15
§2 Exercise # 2; Problems # 3, 5, 9, 12, 15

Assignment #2. *Due on Friday, 23 September.*

§2 Problem # 14

$\boldsymbol{\alpha}$**.** Suppose n is a positive integer and a and b are non-negative integers such that $a = nq_1 + r_1$ with $0 \leq r_1 < n$ and $b = nq_2 + r_2$ with $0 \leq r_2 < n$. If $a+b = nq+r$, where $0 \leq r < n$, and $ab = nq' + r'$, where $0 \leq r' < n$, what are all the possibilities for how q_1 and q_2 are related to q and q' and how r_1 and r_2 are related to r and r'?

Assignment #3. *Due on Friday, 30 September.*

§3 Problem # 9
§4 Problems # 4, 6, 16, 17

Assignment #4. *Due on Friday, 7 October.*

§5 Problems # 12, 16, 18, 20

Assignment #5. *Due on Friday, 14 October.*

§6 Problems # 2, 9, 12
§7 Problems # 5, 6

Assignment #6. *Due on Friday, 21 October.*

Recall from class that Euler's formula for generating amicable pairs of integers is the following rule:

> *Suppose $n > m > 1$ are integers such that $p = 2^m\left(2^{n-m}+1\right) - 1$, $q = 2^n\left(2^{n-m}+1\right) - 1$, and $r = 2^{n+m}\left(2^{n-m}+1\right) - 1$ are all prime numbers. Then $a = 2^n pq$ and $b = 2^n r$ are amicable numbers.*

$\boldsymbol{\beta}$**.** Prove that Euler's formula for generating amicable pairs of integers works.

Extra Assignment #41. *Due on Monday, 31 October.*

Euler gave $p(x) = x^2 + x + 41$ as an example of a polynomial that appears to generate prime numbers for natural number inputs: $p(0) = 41$, $p(1) = 43$, $p(2) = 47$, and so on.

$\boldsymbol{\mu\alpha}$**.** Determine whether $p(n)$ is prime for all $n \in \mathbb{N}$. If so, prove it; if not, prove it and try to find some other polynomial that always gives prime numbers for $n \in \mathbb{N}$.

Assignment #7. *Due on Due on Friday, 4 November.*

§9 Problems # 8, 10, 12, 18

Assignment #8. *Due on Due on Friday, 11 November.*

§10 Problems # 9, 11, 16, 20

Assignment #9. *Due on Due on Friday, 18 November.*

§11 Problems # 6, 8, 11, 16

Assignment #10. *Due on Due on Friday, 25 November.*

§12 Problems # 2, 4, 6(a)(b), 7(a)

Assignment #11. *Due on Due on Friday, 2 December.*

§12 Problems # 8, 10

$\boldsymbol{\gamma}$**.** Consider the polynomial $p(n) = n^2 - 5$. What primes can appear as divisors of $p(n)$ for some $n \geq 3$ and what primes cannot be the divisors of $p(n)$ for any $n \geq 3$?

Optional Assignment #12. *Due on Due on Friday, 9 December.*

§18 Problem # 10
§19 Problem # 7, 10