Trent University

MATH 235H Test

26 February, 2008

Time: 50 minutes

Instructions

- *Show all your work.* Legibly, please!
- *If you have a question, ask it!*
- Use the back sides of the test sheets for rough work or extra space.
- You may use a calculator, and either (both sides of) one 8.5×11 aid sheet or a copy (annotated as you like) of *Formula for Success*.

1. Do any *two* (2) of **a**, **b**, or **c**. *[10 = 2 × 5 each]*

a. Determine whether $\{\, x+2,\ 3x^2+4x+5,\ 6x^2+7x+8,\ 9x^2+10x+11 \,\}$ is a basis for $\mathcal{P}_2$ or not.

b. Determine whether $W = \left\{ \begin{bmatrix} a \\ b \\ 0 \end{bmatrix} \in \mathbb{R}^3 \,\middle|\, a+b=0 \right\}$ is a subspace of $\mathbb{R}^3$ or not.

c. Suppose we keep the usual vector addition in $V = \mathbb{R}^2$, but modify scalar multiplication by setting $c \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} ca \\ -cb \end{bmatrix}$. Show that V is not a vector space.

2. Let $\mathcal{C} = \left\{ \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$.

a. Show that $\mathcal{C}$ is a basis for $\mathbb{R}^3$. *[5]*

b. Find the coordinate vectors relative to the basis $\mathcal{C}$ of $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$, $\begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix}$, and $\begin{bmatrix} 0 \\ 0 \\ 3 \end{bmatrix}$. *[5]*

3. Let the linear tranformation $T : M_{22} \to \mathcal{P}_3$ be defined by

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix} \right) = (a+b)x^3 + (a-b)x^2 + (c-d)x + (c+d)\,.$$

a. Find the kernel, $\ker(T)$, of T. *[5]*

b. Find the range, $\mathrm{ran}(T)$, of T, and determine whether T is invertible. *[5]*

4. Do any *two* (2) of **a**, **b**, or **c**. *[10 = 2 × 5 each]*

a. Suppose U and W are subspaces of a vector space V. Show that $U \cap W = \{\, \mathbf{x} \mid \mathbf{x} \in U \text{ and } \mathbf{x} \in W \,\}$ is also a subspace of V.

b. Suppose V is a vector space and $\mathbf{v} \in V$. Show that $(-1)\mathbf{v} = -\mathbf{v}$.

c. Suppose $T : U \to V$ and $S : U \to V$ are both linear transformation, and define $S + T : U \to V$ by $(S+T)(\mathbf{u}) = S(\mathbf{u}) + T(\mathbf{u})$. Show that $S + T$ is also a linear transformation.

[Total = 40]