Mathematics 235H – Linear algebra II: Vector spaces

TRENT UNIVERSITY, Winter 2008

Quizzes

Quiz #1. Friday, 18 January, 2008. *[10 minutes]*

1. Suppose that V is a vector space, $\mathbf{v} \in V$, and c is a scalar. Show that $c\mathbf{0} = \mathbf{0}$. *[5]*

Quiz #2. Friday, 25 January, 2008. *[10 minutes]*

1. Suppose that $\{ \mathbf{u}, \mathbf{v}, \mathbf{w} \}$ is a linearly independent set of vectors in a vector space V. Show that $\{ \mathbf{u} + \mathbf{v}, \mathbf{v} + \mathbf{w}, \mathbf{u} + \mathbf{w} \}$ is also linearly independent. *[5]*

Quiz #3. Friday, 1 February, 2008. *[10 minutes]*

1. Compute the dimension of the subspace

$$W = \text{Span} \left\{ \begin{bmatrix} 0 & -1 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ -1 & 0 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \right\}$$

of $M_{22}(\mathbb{R})$. *[5]*

Quiz #3. *(Alternate version.) [10 minutes]*

1. Let $W = \{ ax^2 + bx + a \mid a, b \in \mathbb{R} \}$. Verify that W is a subspace of $\mathcal{P}_2$ and find a basis for W. *[5]*

Quiz #4. Friday, 8 February, 2008. *[10 minutes]*

1. Suppose U and V are vector spaces using the same scalars, and that $U = \text{Span}\{\mathbf{b}\}$ for a vector $\mathbf{b} \in U$. Show that a function $T : U \to V$ such that $T(c\mathbf{b}) = cT(\mathbf{b})$ for any scalar c must be a linear transformation. *[5]*

Quiz #5. Friday, 15 February, 2008. *[10 minutes]*

1. Determine whether the linear transformation $T : \mathbb{R}^3 \to \mathcal{P}_2$ given by

$$T\left(\begin{bmatrix} a \\ b \\ c \end{bmatrix} \right) = (a - b)x^2 + (b - c)x + (c - a)$$

is one-to-one or not. *[5]*

Quiz #6. Friday, 7 March, 2008. *[10 minutes]*

1. Suppose V is a vector space with an inner product $\langle \, , \, \rangle$. Verify that for any vector $\mathbf{v} \in V$ and scalar c, $\|c\mathbf{v}\| = |c| \, \|\mathbf{v}\|$. *[5]*

Quiz #7. Friday, 14 March, 2008. *[10 minutes]*

1. Use Gram-Schmidt orthogonalization to modify the basis $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \end{bmatrix} \right\}$ of $\mathbb{R}^2$ into a basis which is orthonormal with respect to the inner product $\left\langle \begin{bmatrix} a \\ b \end{bmatrix}, \begin{bmatrix} c \\ d \end{bmatrix} \right\rangle = 2ac + bd$ on $\mathbb{R}^2$. *[5]*

Quiz #8. Wednesday, 19 March, 2008. *[10 minutes]*

1. Suppose V is a finite-dimensional vector space using the real numbers as scalars. Define an inner product on V. *[5]*

Quiz #9. Friday, 28 March, 2008. *[10 minutes]*

1. Let $\left\| \begin{bmatrix} a \\ b \end{bmatrix} \right\| = a^2 + b^2$. Determine if this is a norm on $\mathbb{R}^2$ or not. *[5]*

Quiz #9. *(Alternate version. [10 minutes]*

1. Let $\|p(x)\| = |p(0)|$. Determine if this is a norm on $\mathcal{P}_1$ or not. *[5]*

Quiz #10. Friday, 4 April, 2008. *[10 minutes]*

1. Find the vector closest to $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ in the subspace $W = \text{Span}\left\{ \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \right\}$ of $\mathbb{R}^3$. *[5]*