

Mathematics 235H – Linear algebra II: Vector spaces

TRENT UNIVERSITY, Winter 2008

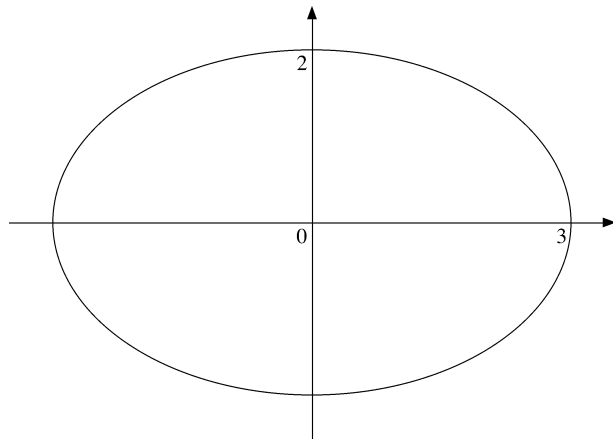
Assignment #8

Due on π -day, 14 March, 2008.

Areas via linear transformations

1. Determine the area of the ellipse $\frac{x^2}{9} + \frac{y^2}{4} = 1$ using linear algebra. [10]

Hint: Find a linear transformation from \mathbb{R}^2 to \mathbb{R}^2 that transforms the unit circle into the given ellipse, and then determine how that linear transformation changes areas. You may use the fact that the area of a circle of radius r is πr^2 .



Assignment #9

Due on Tuesday, 18 March, 2008.

Vector spaces of linear transformations

Recall from class that if U and V are vector spaces (using the same set of scalars), then

$$L(U, V) = \{ T \mid T \text{ is a linear transformation } U \rightarrow V \}$$

is itself a vector space (using the same scalars as U and V) when vector addition is defined by

$$(T + S)(\mathbf{u}) = T(\mathbf{u}) + S(\mathbf{u})$$

and scalar multiplication is defined by

$$(cT)(\mathbf{u}) = cT(\mathbf{u}).$$

1. Suppose that $\dim(U) = 3$ and $\dim(V) = 4$. What is $\dim(L(U, V))$? Explain why! [5]
2. Determine whether each of the following sets is a subspace of $L(U, V)$ or not.
 - a. $W = \{ T \in L(U, V) \mid T \text{ is one-to-one} \}$ [1]
 - b. $W = \{ T \in L(U, V) \mid T \text{ is onto} \}$ [1]
 - c. $W = \{ T \in L(U, V) \mid T \text{ is invertible} \}$ [1]
 - d. $W = \{ T \in L(U, V) \mid T \text{ is not invertible} \}$ [2]