

Mathematics 235H – Linear algebra II: Vector spaces

TRENT UNIVERSITY, Winter 2008

Assignment #11

Due on Friday, 4 April, 2008.

Moving triangles in perspective

Two triangles $\triangle ABC$ and $\triangle XYZ$ are in *perspective* from a point P if the lines AX , BY , and CZ all pass through P . We'll be looking at moving triangles in perspective from the origin in \mathbb{R}^3 using linear transformations. In what follows let $\triangle ABC$ have vertices $A = (1, 0, 0)$, $B = (0, 1, 0)$, and $C = (0, 0, 1)$, and let $\triangle XYZ$ have vertices $X = (2, 0, 0)$, $Y = (0, 3, 0)$, and $Z = (0, 0, 4)$. It's obvious that $\triangle ABC$ and $\triangle XYZ$ are in perspective from the origin.

1. Find an invertible linear transformation from \mathbb{R}^3 to \mathbb{R}^3 that moves $\triangle ABC$ to $\triangle XYZ$. [3]
2. In general, if $\triangle DEF$ and $\triangle RST$ are in perspective from the origin and are not in the same plane, show there must be an invertible linear transformation from \mathbb{R}^3 to \mathbb{R}^3 that moves $\triangle DEF$ to $\triangle RST$. [3]
3. Find the coordinates of the points where the line AB meets the line XY , the line AC meets the line XZ , and the line BC meets the line YZ , respectively. Is there anything special about these points? [4]