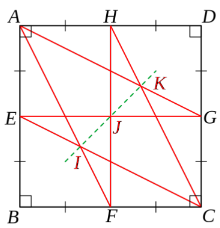
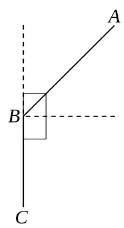
Mathematics 2260H – Euclidean Geometry
Trent University, Winter 2026

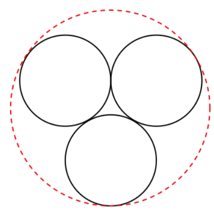
Take-Home Final Examination
Due on Friday, 17 April.*

Instructions: Do both of parts **I** and **II**, and, if you wish, part **III** as well. Show all your work. You may use your textbooks and notes, as well as any handouts and returned work, from this and any other courses you have taken or are taking now. You may also ask the instructor to clarify the statement of any problem, and use calculators or computer software to do numerical computations and to check your algebra. However, *you may not consult any other sources, nor consult or work with any other person on this exam.* Please draw any relevant diagram(s) for each problem that you choose to do.

Part I. Do any *four* (4) of problems **1** – **5**. *[40 = 4 × 10 each]*







1. Suppose $ABCD$ is a square and E, F, G, and H are the midpoints of AB, BC, CD, and DA, respectively. Let $I = AF \cap CE$, $J = EG \cap FH$, and $K = AG \cap CH$. Show that I, J, and K are collinear. *[Not a theorem of Pappus', as far as I know.]*

2. Give a complete straightedge-and-compass construction of an angle of three quarters of a straight angle, and explain why it works.

3. Three circles of radius 1 are all tangent to one another, and a fourth circle containing all three circles is also tangent to all of them. Determine the radius of the fourth circle.

4. Use Menelaus' Theorem to prove Ceva's Theorem.

5. Let H be the orthocentre of $\triangle ABC$. Show that C is the orthocentre of $\triangle ABH$.

Part II. Do any *four* (4) of problems **6** – **13**. *[40 = 4 × 10 each]*

6. Suppose A, B, and C are distinct points on a line ℓ, and A', B', and C' are distinct points not on ℓ such that the points $D = AB' \cap A'B$, $E = AC' \cap A'C$, and $F = BC' \cap B'C$ exist and are collinear. Show that A', B', and C' are also collinear. *[This is a converse of sorts to Desargues' Theorem.]*

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* You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper as soon as you can.

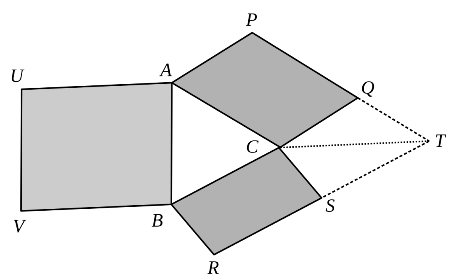
7. Suppose $\triangle ABC$ is isosceles with $|AB| = |AC|$, and D is a point on the same side of BC as A such that $\angle BAC = 2\angle BDC$. Show that A is the circumcentre of $\triangle DBC$.

8. Suppose the line segment AB is parallel to, but not part of, the infinite line ℓ. Show that there is an unique circle passing through A and B that is tangent to the line ℓ.

9. Suppose P, Q, and R are the points on BC, AC, and AB, respectively, where the angle bisectors from A, B, and C, respectively, meet the sides of $\triangle ABC$. Let S be the point on the extension of AB such that CS is perpendicular to CR. Show that P, Q, and S are collinear.

10. Suppose $ABCD$ is a cyclic quadrilateral, *i.e.* A, B, C, and D are points on a circle in counterclockwise order. Show that if we join each of A, B, C, and D to the orthocentre of the triangle formed by the other three, then the resulting line segments all intersect in a common midpoint M.

11. Suppose that the incircle of $\triangle ABC$ touches AB at Z, BC at X, and AC at Y. Show that AX, BY, and CZ are concurrent.



12. Suppose we are given $\triangle ABC$ and parallelograms $ACQP$ and $BCSR$. Let T be the point where PQ intersects RS. Connect C to T, and let $ABVU$ be the paralelogram such that $AU \parallel TC \parallel BV$ and $|AU| = |TC| = |BV|$. Show that the area of $ABVU$ is equal to the sum of the areas of $ACQP$ and $BCSR$. *[A theorem of Pappus' generalizing the Pythagorean Theorem.]*

13. Give a dissection of a square for which the pieces can be reassembled into a regular hexagon.

[Total = 80]

Part III. Bonus!

▲. Write an original poem about Euclidean geometry. *[1]*

■. What is the maximum number of points which are all the same distance from each other that can be found in the Euclidean plane? Explain why. *[1]*

I HOPE YOU'VE ENJOYED THIS COURSE!
NOW ENJOY THE SUMMER!