## Mathematics 2260H – Geometry I: Euclidean geometry TRENT UNIVERSITY, Winter 2025

## Assignment #6 The Flutterby Butterfly Theorem Due on Friday, 28 February.\*

The following theorem, which seems to have been named after the diagram, first appeared as a problem posed by William Wallace (1768-1843) in 1803 in *The Gentleman's Mathematical Companion*. Wallace was a Scottish mathematician and astronomer who was a student of John Playfair (1748-1819), after whom Playfair's Postulate is named.

BUTTERFLY THEOREM. Suppose PQ is a chord of a circle, M is the midpoint of PQ, AB and CD are two other chords of the circle that pass through M, and suppose AD and BC meet PQ in X and Y respectively. Then M is the midpoint of XY.



**1.** Prove the Butterfly Theorem. [10]

*Hint:* Draw perpendiculars from each of X and Y to each of AB and CD. There will now be wealth of similar triangles in the picture, with various length relationships to exploit algebraically. The Inscribed Angle Theorem (question **2** on Assignment #3) may be of use in getting all the similarities available.

<sup>\*</sup> Please submit your solutions, preferably as a single pdf, via Blackboard's Assignments module. If that fails, please submit them to the instructor on paper or via email to sbilaniuk@trentu.ca as soon as you can.