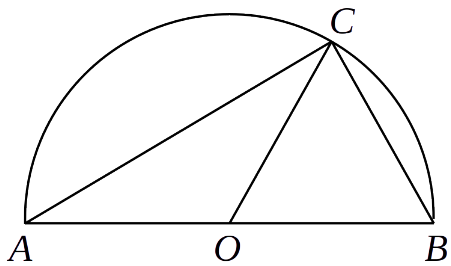
Mathematics 2260H – Euclidean Geometry
TRENT UNIVERSITY, Winter 2025
Solutions to Assignment #2
Triangles and Angles

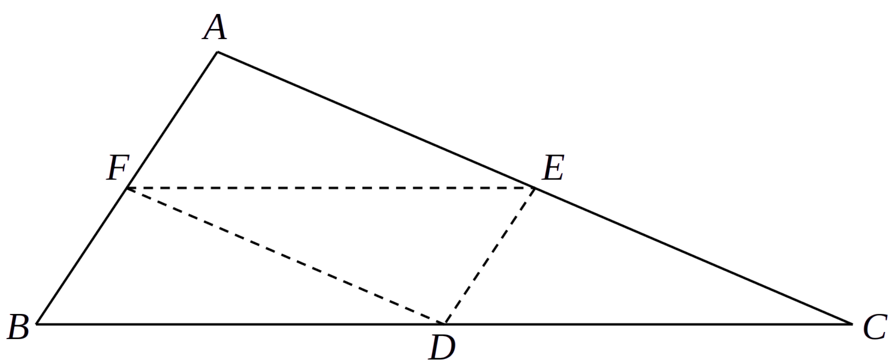
Please read, or at least skim, through the handout *Similar Triangles and Similarity Criteria* before doing this assignment. In particular, note the definition of similarity and the criteria for establishing the similarity of triangles. For this assignment you may freely use the fact that the interior angles of any triangle add up to two right angles, even though this fact is equivalent to the parallel postulate.



1. Suppose that AB is the diameter of a circle, O is the centre of the circle (and hence also the midpoint of AB), and C is some point on the circle different from both A and B. Show that $\angle BOC = 2\angle BAC$. *[5]*

SOLUTION. Since OA, OB, and OC are all radii of the same circle, they all have equal length. Thus $\triangle AOC$ and $\triangle BOC$ are both isosceles, and so, by Proposition I-5, $\angle OAC = \angle OCA$ and $\angle OBC = \angle OCB$. Since the sum of the internal angles of any triangle is equal to the sum of two right angles, *i.e.* to π if we measure the angles in radians, we have $\angle OAC + \angle OCA + \angle AOC = \pi$ and $\angle OBC + \angle OCB + \angle BOC = \pi$. By Proposition I-13, we have that $\angle AOC + \angle BOC = \pi$. Note also that $\angle OAC = \angle BAC$ because it is the same angle. Putting *some* of this together:

$$\begin{aligned} \angle BOC &= \pi - \angle AOC = \pi - (\pi - \angle OAC - \angle OCA) \\ &= \angle OAC + \angle OCA = 2\angle OAC = 2\angle BAC \qquad \square \end{aligned}$$



2. Suppose that D, E, and F are the midpoints of the sides BC, AC, and AB, respectively, of $\triangle ABC$. Show that $\triangle DEF \sim \triangle ABC$. *[5]*

SOLUTION. Since $\angle FAE = \angle BAC$, being the same angle, while $\frac{|AF|}{|AB|} = \frac{1}{2} = \frac{|AE|}{|AC|}$, because F and E are the midpoints of AB and AC, respectively, it follows by the Side-Angle-Side similarity criterion that $\triangle AFE \sim \triangle ABC$. From this, in turn, it follows that $\frac{|FE|}{|BC|} = \frac{|AF|}{|AB|} = \frac{|AE|}{|AC|} = \frac{1}{2}$.

Similarly*, since $\angle FBD = \angle ABC$, being the same angle, while $\frac{|AF|}{|AB|} = \frac{1}{2} = \frac{|BD|}{|BC|}$, because F and D are the midpoints of AB and BC, respectively, it follows by the Side-Angle-Side similarity criterion that $\triangle FBD \sim \triangle ABC$. From this, in turn, it follows that $\frac{|FD|}{|AC|} = \frac{|AF|}{|AB|} = \frac{|BD|}{|BC|} = \frac{1}{2}$.

Similarly* (again), since $\angle FBD = \angle ACB$, being the same angle, while $\frac{|AE|}{|AC|} = \frac{1}{2} = \frac{|DC|}{|BC|}$, because E and D are the midpoints of AC and BC, respectively, it follows by the Side-Angle-Side similarity criterion that $\triangle EDC \sim \triangle ABC$. From this, in turn, it follows that $\frac{|ED|}{|AB|} = \frac{|AE|}{|AC|} = \frac{|BD|}{|BC|} = \frac{1}{2}$.

Since $\frac{|FE|}{|BC|} = \frac{|FD|}{|AC|} = \frac{|ED|}{|AB|} = \frac{1}{2}$, it follows by the Side-Side-Side Similarity criterion that $\triangle DEF \sim \triangle ABC$. ■

* Pun intended ...