

Mathematics 2260H – Geometry I: Euclidean Geometry

TRENT UNIVERSITY, Winter 2024

Assignment #3

The Elliptic Plane via Linear Algebra

With a side of the Lord of the Rings

Due just before midnight on Friday, 9 February.*

Consider the geometry \mathcal{G} defined as follows:

1. The points of \mathcal{G} are the one-dimensional subspaces (*i.e.* lines through the origin) of \mathbb{R}^3 .
2. The lines of \mathcal{G} are the two-dimensional subspaces (*i.e.* planes through the origin) of \mathbb{R}^3 .
3. Point P of \mathcal{G} is on line ℓ of \mathcal{G} if the subspace P is contained in the subspace ℓ .
4. The (smaller) angle between two lines of \mathcal{G} is the (smaller) angle between them as planes of \mathbb{R}^3 .
5. The (smaller) distance between two points of \mathcal{G} is the (smaller) angle between them as lines of \mathbb{R}^3 .

1. Explain which of Euclid's Postulates hold in \mathcal{G} and which do not. [7]

Consider the geometry described in the following poem:

If the Númenoreans had been mathematicians[†]

*Tall ships and tall kings
Three times three,
What brought them from the foundered land
Over the flowing sea?
Seven points and seven lines
In one geometry:
Every point met three lines,
Every line met points three,
Every pair of points connected,
Every line pair intersected.
Fano found it, Fano named it.*

With apologies to J.R.R. Tolkien . . .

2. Draw the geometry described in the poem as best you can. [3]

* You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper.

[†] . . . the rhyme of lore recited by Gandalf to Pippin in their ride from Rohan to Gondor – “Tall ships and tall kings/ Three times three,/ What brought them from the foundered land/ Over the flowing sea?/ Seven stars and seven stones/ And one white tree.” – might have been a little different.

The Fano plane is named after the Italian mathematician Gino Fano (1871-1952), one of the pioneers of finite geometry. If we were to add him to this alternate version of Tolkien's *legendarium*, we could make him another half-sibling of Fëanor, the maker of the Silmarils and the palantíri.