

# Mathematics 2260H – Geometry I: Euclidean Geometry

TRENT UNIVERSITY, Winter 2024

## Assignment #3

### More Possible Congruence Criteria

*Due\* just before midnight on Friday, 2 February.*

So far, we have encountered three congruence criteria for triangles:

1. (*Side-Angle-Side*) Given triangles  $\triangle ABC$  and  $\triangle DEF$ , if  $|AB| = DE$ ,  $\angle BAC = \angle EDF$ , and  $|AC| = |DF|$ , then  $\triangle ABC \cong \triangle DEF$ . (Proposition I-4)
2. (*Side-Side-Side*) Given triangles  $\triangle ABC$  and  $\triangle DEF$ , if  $|AB| = DE$ ,  $|BC| = |EF|$ , and  $|AC| = |DF|$ , then  $\triangle ABC \cong \triangle DEF$ . (Proposition I-8)
3. (*Angle-Side-Angle*) Given triangles  $\triangle ABC$  and  $\triangle DEF$ , if  $\angle ABC = \angle DEF$ ,  $|BC| = |EF|$ , and  $\angle ACB = \angle DFE$ , then  $\triangle ABC \cong \triangle DEF$ . (Proposition I-26)

What other combinations of three of the three internal angles and the three sides of a triangle yield valid congruence criteria for triangle?

1. (*Angle-Angle-Angle*) Given triangles  $\triangle ABC$  and  $\triangle DEF$ , suppose  $\angle ABC = \angle DEF$ ,  $\angle BAC = \angle EDF$ , and  $\angle ACB = \angle DFE$ . Must we have  $\triangle ABC \cong \triangle DEF$ ? Prove it or give a counterexample. [1]
2. (*Angle-Angle-Side*) Given triangles  $\triangle ABC$  and  $\triangle DEF$ , suppose  $\angle ABC = \angle DEF$ ,  $\angle BAC = \angle EDF$ , and  $|AC| = |DF|$ . Must we have  $\triangle ABC \cong \triangle DEF$ ? Prove it or give a counterexample. [3]
3. (*Side-Side-Angle*) Given triangles  $\triangle ABC$  and  $\triangle DEF$ , suppose  $|AB| = DE$ ,  $|BC| = |EF|$ , and  $\angle ACB = \angle DFE$ . Must we have  $\triangle ABC \cong \triangle DEF$ ? Prove it or give a counterexample. [4]
4. In one of **1-3**, the criterion fails for certain values of (one of ) the angle(s) involved, but works for other possible values. Which criterion is it and what are the values? [2]

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\* You should submit your solutions via Blackboard's Assignments module, preferably as a single pdf. If submission via Blackboard fails, please submit your work to your instructor by email or on paper.