Lecture 9

Thursday, January 25, 2024 10:39 AM

What happens if the parallel axiom fails?

(A side for the main course...)

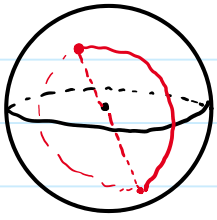
How could this fail?

Failure mode 1: there is more than one line through the point parallel to the given line

Failure mode 2: there is no line through the point parallel to the given line

Failure mode 2:

A model of such an "elliptic" geometry:



→ "points" are opposite pairs of points on the sphere.

→ "lines" are the great circles of the sphere

Some of the internal of a triangle $> 2\pi$

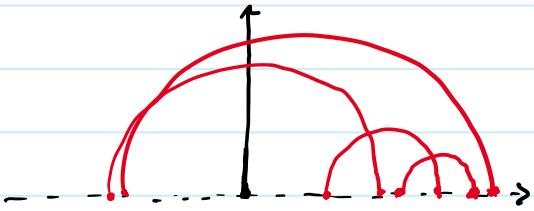
Area of ΔABC (if you have one) = (sum of the internal angles $- \pi$)

Failure mode 1: Suppose there are more than one parallel line.

"hyperbolic geometry"

"hyperbolic geometry"

Poincaré half-plane model



→ "points" are the points of $\mathbb{R}$ w $y>0$.

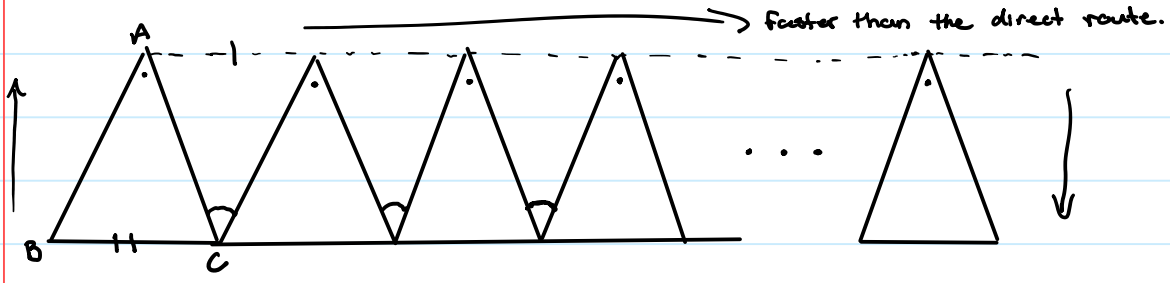
→ "lines" are upper semicircles w centres on the x-axis.

Here the sum of the internal angles of a triangle is always less than $2\angle$.

Area of $\triangle ABC = \pi$ − sum of internal angles

dk

Why Postulate 2 must be modified if there are no parallel lines...



$\Rightarrow \angle < \Delta \Rightarrow$ --+-- < ——