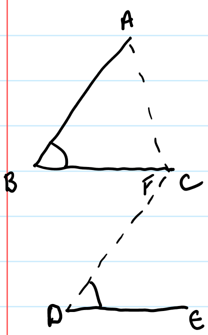
Lecture 8

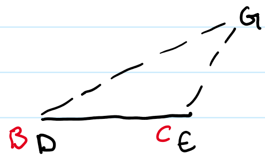
Thursday, January 25, 2024 9:01 AM

I-23: Given a rectilinear angle $\angle ABC$ and a line segment DE construct F such that $\angle FDE = \angle ABC$

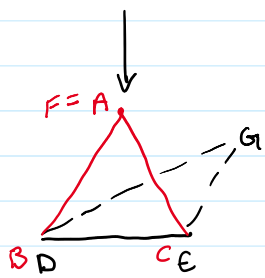


Proof: connect A to C to make $\triangle ABC$.
Pick a point G (not on any extension of) DE

connect G to D and E to make $\triangle DEG$.

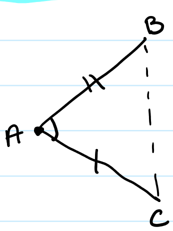


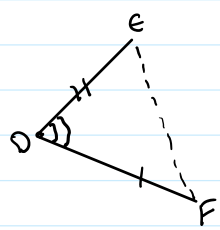
Apply $\triangle ABC$ to $\triangle DEG$ so that B is on D and BC lies along DE and A is on the same side of DE as G.



Then let $F = A$ and obviously $\angle FDE = \angle ABC$ //

I-24 and 25:



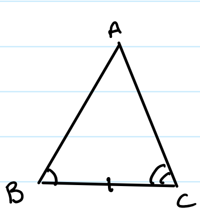


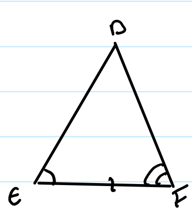
Given $|AB| = |DE|$ and $|AC| = |DF|$

Then say $\angle EDF < \angle BAC \iff |EF| < |BC|$

$$|EF|^2 = |DE|^2 + |DF|^2 - 2|DE||DF|\cos(\measuredangle)$$ cosine Law!

I-26: (Angle-Side-Angle congruence criterion)

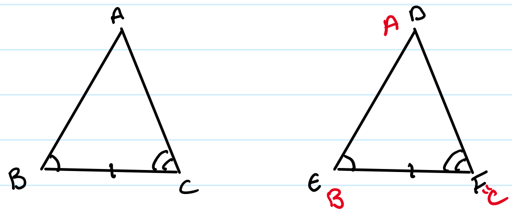




if $|BC| = |EF|$ and $\angle ABC = \angle DEF$, and $\angle ACB = \angle DFE$, then $\triangle ABC \cong \triangle DEF$

Proof: Apply $\triangle ABC$ to $\triangle DEF$ so that B is on E and BC is along EF and A is on the same side of EF as D.

since we have $|BC| = |EF|$, must have



Since we have $|BC| = |EF|$, must have C is on F. ($F = C$)

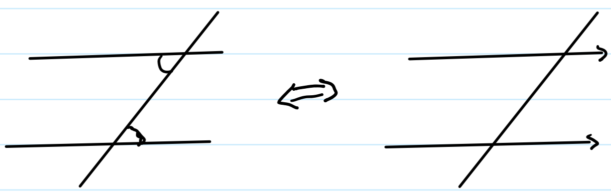
Since $\angle DEF = \angle ABC$ and BC coincides with EF and A and D are on the same side of EF.

We must have BA lie along ED. Similarly CA lies along FD.

It follows that $A = BA \cap CA$ (intersect)
$= ED \cap FD$ $\quad \therefore A = D$ (A is on D)
$= D$

$\therefore$ Since A is on D, B is on E, and C is on F,
$\triangle ABC \cong \triangle DEF$

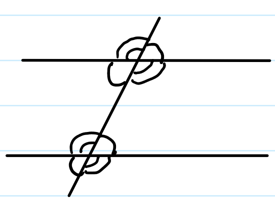
I-27 and 28: (Z-theorem)



A line falling across two other lines makes equal alternate angles iff the two other lines are parallel.

Fwd dir.
$\Rightarrow$ does <u>not</u> need Post. 5 ,
(I-27)

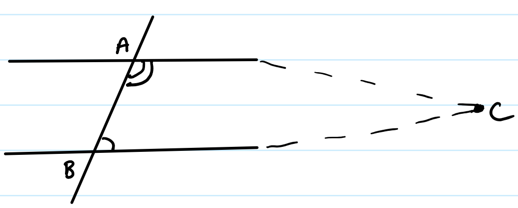
backward dir.
$\Leftarrow$ <u>does</u> need Postulate. 5
(I-28)

I-27: 

If the angles are equal, then the lines do not intersect.

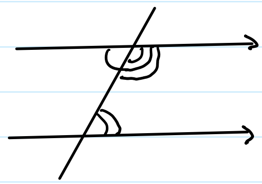
Proof: Assume by way of contradiction that the lines intersect.

$\Rightarrow \angle + \angle = 2\angle$
$\Rightarrow \angle + \angle < 2\angle$ $\otimes$ //

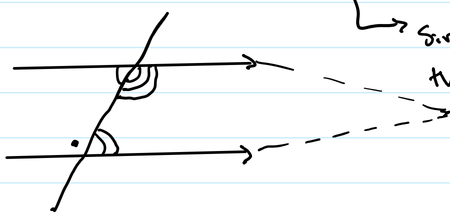


I-28:

To show $\angle = \angle$



Proof! we know by postulate 5 that if $\angle + \angle < 2\llcorner$, then the lines will cross on that side.



Similarly if $\angle + \angle < 2\llcorner$ they'd cross on that side.

If $\angle + \angle$ can't be $< 2\llcorner$, and can't be $> 2\llcorner$, then $\angle + \angle = 2\llcorner$.

But $\angle + \angle = 2\llcorner$, so $\angle = \angle$. //