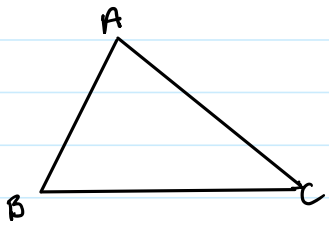
Lecture 7

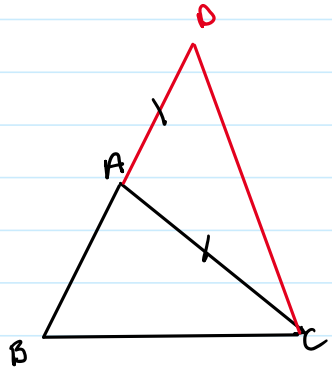
Tuesday, January 23, 2024 8:49 AM

I-20: In any triangle, the sum of any two sides is greater than the remaining side.



$|AB| + |AC| > |BC|$

Proof: Extend AB past A to D s.t. $|AD| = |AC|$



connect C to D to make $\triangle DBC$ and $\triangle ACD$.

if we can show that $\angle DCB > \angle BDC$, then

$|BC| < |BD| = |AB| + |AD| = |AB| + |AC|$

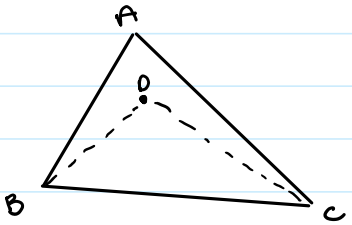
But $\angle DCB = \angle DCA + \angle ACB$

$= \angle CDA + \angle ACB$

$\angle BDC$

$= \angle BDC + \angle ACB > \angle BDC$

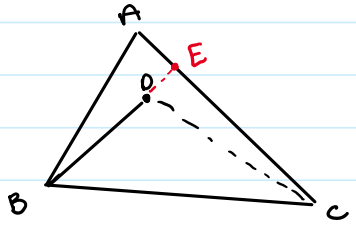
I-21: Suppose the point D is inside $\triangle ABC$



if we connect D to B and C to make $\triangle DBC$, then $\angle BDC > \angle BAC$

Then $|AB| > |DB|$, and $|AC| > |DC|$.

Proof:



Extend BD past D to E on AC.

Then $|BE| < \overbrace{|BC| + |CE|}^{\text{ignore}}$

$< |AB| + |AE|$ → use this one

↳ $|BE| < |AB| + |AE|$, so $|BE| + |EC| < |AB| + |AE| + |EC|$

$\hookrightarrow |BE| < |AB| + |AE|$, so $|BE| + |EC| < |AB| + |AE| + |EC|$

* add $|EC|$ to both sides

$\Rightarrow |BE| + |EC| < |AB| + |AE| + |EC|$

$|BD| + |DE| + |EC| = |BD| + |DC|$

$|AB| + |AC|$

$\}$ $|BD| + |DC| < |AB| + |AC|$ ✱ This is what Euclid tries to prove!

It remains to show that $\angle BAC < \angle BDC$:

$\angle BDC > \angle DEC = \angle BEC$

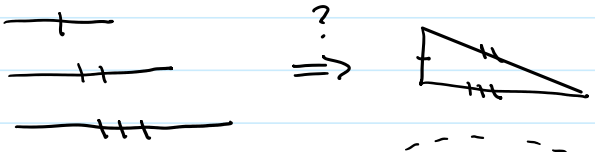
$\angle BDC$: external angle for ΔEDC

$\angle DEC$: external angle for ΔABE

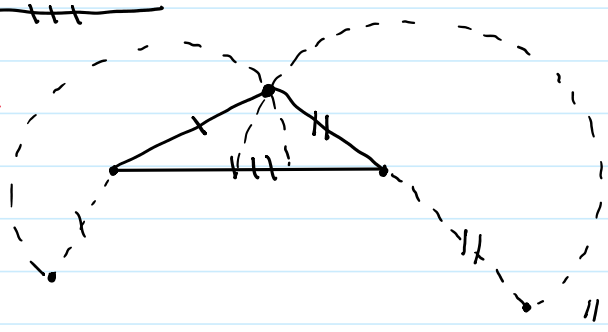
$\}$ So, $\angle DEC > \angle ABE = \angle ABC$!

$\therefore \angle BAC < \angle BDC$ //

I-22: Given three line segments such that any two add up to more than the third, we can use these line segments as the sides of the same triangle.



Proof:



draw largest line segment, connect other two line segments. use postulate 3.