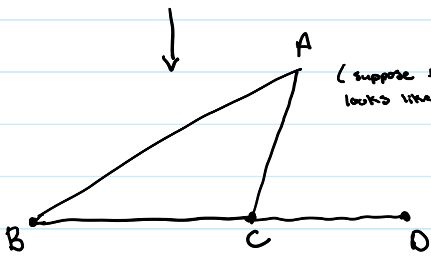
Lecture 6

Thursday, January 18, 2024 11:49 AM

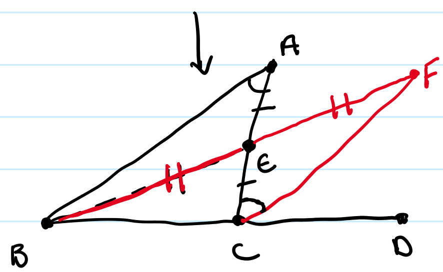
I-16: Suppose we extend side BC of $\triangle ABC$ past C to D.
Then $\angle BAC < \angle ACD$ and $\angle ABC < \angle ACD$. (show)

Proof:



(suppose triangle looks like this)

Find middle pt of AC. Let this pt be 'E', connect B to E and extend to F. $|BE| = |FE|$.



Connect F to C. Consider $\triangle EFC$ and $\triangle EBA$. These are congruent by S-A-S (side-Angle-side)

$|BE| = |FE|$ (by construction)

$|CE| = |AE|$ (mid pt.)

$\angle AEB = \angle CEF$ (by opposite angle theorem)

$\therefore \triangle EFC \cong \triangle EBA$

$\therefore \angle BAE = \angle FCE$

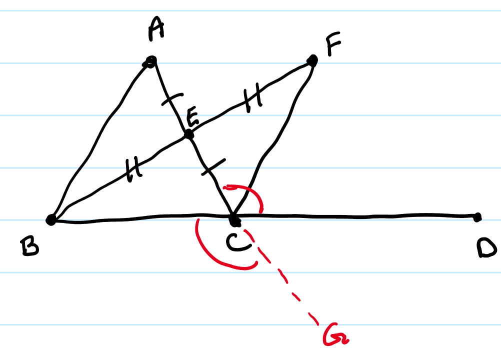
$$\begin{aligned} \angle ACD &= \angle ACF + \angle FCD \\ &= \angle FCE + \angle FCD \\ &= \angle BAE + \angle FCD \\ &= \angle BAC + \angle FCD \end{aligned}$$

$\therefore \angle ACD > \angle BAC$. //

What about $\angle ABC < \angle ACD$?

→ Extend AC past 'C' to some pt 'G'

→ by opposite angle theorem $\angle ACD = \angle BCG$



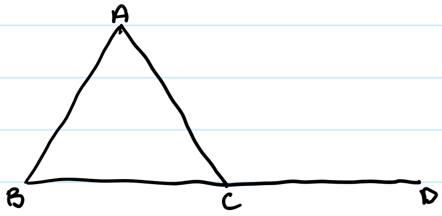
But $\angle ABC$ is to $\angle BCG$ as $\angle BAC$ was to $\angle ACD$

... $\angle ABC < \angle BCG = \angle ACD$

But $\angle ABC$ is to $\angle BCG$ as $\angle BAC$ was to $\angle ACD$

By the same argument ("mutatis mutandis") $\angle ABC < \angle BCG = \angle ACD$ //

I-17:



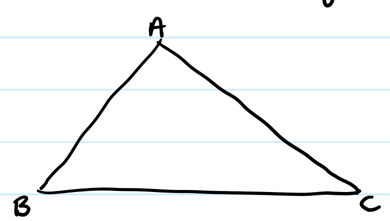
<u>To show:</u> $\angle ABC + \angle BAC < 2\llcorner = \frown$

Proof: $\angle BCA + \angle ACD = 2\llcorner = \frown$

since B, C, D are on the same straight line.

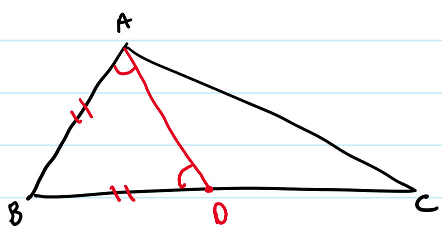
$$(\angle ABC + \angle BCA) < (\underbrace{\angle ACD + \angle BCA}_{= \, 2\llcorner})$$ by I-16 //

I-18: In any triangle the greater side subtends the greater angle.



$|AB| < |BC| \Rightarrow \angle ACB < \angle BAC.$

Proof:



$|AB| < |BC|.$

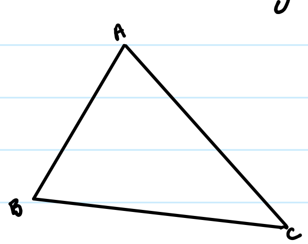
$|BD| = |AB|$ → d is between B & C

we know $\angle BAD < \angle BAC$

isosceles ⇒ $\angle BAD = \angle BDA > \angle ACD$ → external $\angle$ of $\triangle ACD$

$\angle ACD = \angle ACB$

I-19: In any triangle the greater angle is subtended by the greater side.



$\angle ACB < \angle BAC \Rightarrow |AB| < |BC|$

Proof: This time we know angle relationship, want to know side relation.

(By contradiction) Suppose $\angle ACB < \angle BAC$, but $|AB| \geq |BC|$.

If $|AB| = |BC|$, we would have $\triangle ABC$ as an isosceles. So we'd have

$\angle BAC = \angle ACB$ $\otimes$ contradiction!

If $|AB| > |BC|$, by I-18 $\angle ACB > \angle BAC$. $\otimes$ contradiction! //