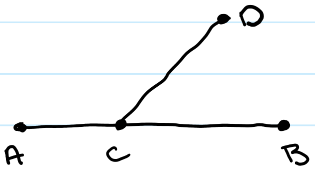
Lecture 5

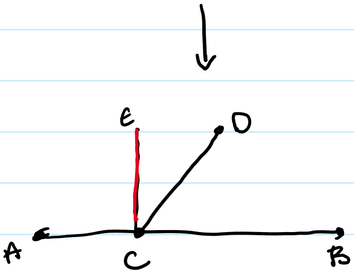
Thursday, January 18, 2024 8:39 AM

I-13: (proposition)



If DC meets AB at C (between A & B) then $\angle ACD + \angle DCB = \underset{\overset{\nearrow}{90^\circ}}{2\llcorner} = \underset{\overset{\nearrow}{180^\circ}}{\frown}$.

Proof:



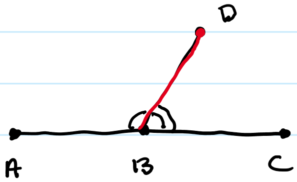
make a right angle $\angle ACE$ at C.

Then $\angle ECB$, is also a right angle.

But $\underset{\frown}{\angle ACB} = \angle ACE + \angle ECB = 2\llcorner$

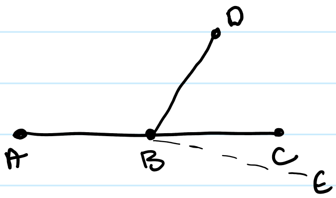
$= \angle ACD + \angle DCB$. ‖

I-14:



If $\angle ABD + \angle DBC = 2\llcorner = \frown$, then AB & BC are parts of the same straight line

Proof: suppose $\angle ABD + \angle DBC = 2\llcorner$



Suppose by way of contradiction that C is not on any extension of AB.

Extend AB past B (assume C is on the other side of DB from A) to a point E such that $|BE| > |BC|$.

By I-13, $\angle ABD + \angle DBE = 2\llcorner$

‖

$\underbrace{\angle ABD + \angle BDC + \angle CBE}_{= 2\llcorner}$

Sum $> 2\llcorner$ ⊗

→ So $2\llcorner > 2\llcorner$, violating postulate I-4

Hence A,B,C are on the same straight line

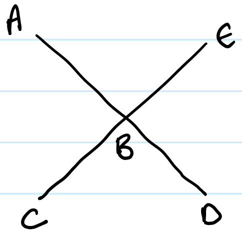
I-15: (opposite angle theorem)

If two straight lines cross eachother, then the opposite angles are equal.

Proof:

Suppose AD crosses CE at B.

Proof:



Suppose AD crosses CE at B.

Show: $\angle ABC = \angle CBD$ & $\angle ABC = \angle DBE$

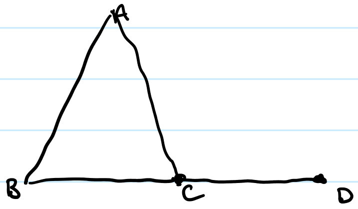
↑ similarly for these

$\angle ABE + \angle EBD = 2\llcorner$ by I-13

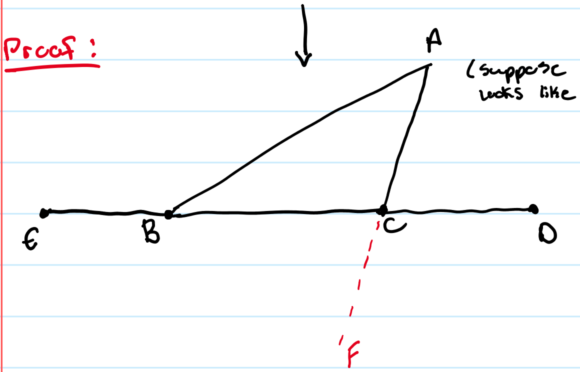
$\angle CBD + \angle \overset{\shortparallel}{D}BE = 2\llcorner$ by I-13

$\therefore \angle ABE = 2\llcorner - \angle EBD = \angle CBD$,,

I-16: Suppose we extend side BC of $\triangle ABC$ past C to D.
Then $\angle BAC < \angle ACD$ and $\angle ABC < \angle ACD$.



Proof:



(suppose triangle looks like this)

Next time!