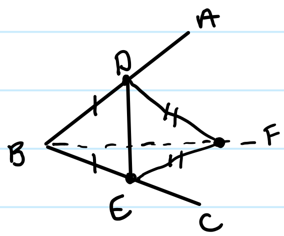
Lecture 4

Thursday, January 18, 2024 8:39 AM

I-9: Given an angle, cut it in half



So bisecting an angle can be done w (with) compass and straightedge trisecting usually can't

Proof: 1) draw a circle w centre B intersecting AB and AC at D & E respectfully

2) Construct an equilateral triangle ΔDEF

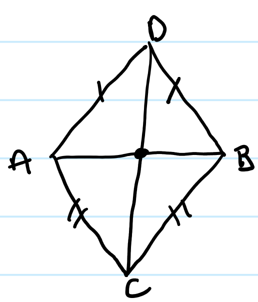
3) connect B and F

4) $|BD| = |BE|$ (radii) and $|DF| = |EF|$ (sides of equilateral triangle and $|BE| = |BF| \Rightarrow \Delta BDF \cong \Delta BEF$ by S-S-S, I-8)

$\therefore \angle ADF = \angle BDF = \angle EBF = \angle CBF$

I-10: Given a line segment, cut it in half

Proof: Given AB find C between A and B so that $|AC| = |BC|$



1) construct equilateral triangle w base AB on both sides of AB say ΔADB and ΔAEB

2) connect D and E and let C be the intersection of AB and DE

Claim: $|AC| = |BC|$

First $\Delta DAE \cong \Delta DBE$ by S-S-S b/c (because) $|DA| = |DB|$, $|AE| = |BE|$, $|DE| = |DE|$

Second $\Delta DAE \cong \Delta DBE$

$\Rightarrow \angle ADE = \angle BDE$

$\angle ADE = \angle ADC$, $\angle BDE = \angle BDC$

$\Rightarrow \angle ADE = \angle BDE$

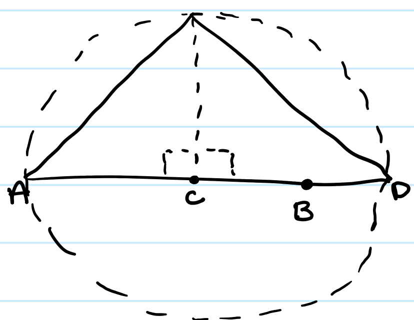
$\angle ADC \qquad \angle BDC$

Third $|AD| = |BD|$ and $|DC| = |DC|$ and $\angle ADC = \angle BDC$ so $\triangle ADC \cong \triangle BDC$ by s-A-s and prop. I-4

Fourth $\triangle ADC \cong \triangle BDC$

$\Rightarrow |AC| = |BC|$ //

I-11: At a given pt on a line, construct a line perpendicular to the given one



Proof: 1) Draw a circle w radius AC and centre C (extend AB past B until it meets the circle) meeting CB at D

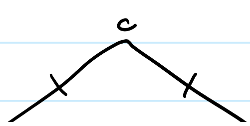
2) construct an equilateral triangle $\triangle ADE$ on AD. connect E to C.

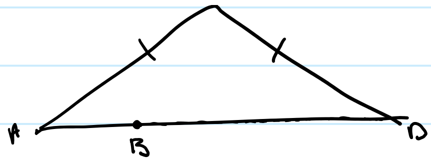
3) $\triangle AEC \cong \triangle DEC$ since $|AE| = |DE|$ (sides of an equilateral triangle) and $|EC| = |EC|$ and $|CA| = |CD|$ (radii)

$\therefore \angle ACE = \angle DCE$ but $\angle ACE + \angle DCE = \angle ACD$ = straight angle

$\therefore \angle ACE = \angle DCE$ are right angles by definition

I-12: Given a line AB and a pt C (not any extension of AB) construct a line through C perpendicular to AB.





Proof: 1) Join C to A and draw a radius w/ circle w/ radius CA and centre C. If the circle only touches (any extension of) AB at A, then pick a different pt of AB to draw a radius to.

2) So we need only consider the case where the circle intersects AB (or an extension) at two pts A and D

3) connect C to D, then $|AC| = |DC|$ (radii)

4) Let E be the midpt. of AD, so $|AE| = |DE|$ since $|CE| = |CE|$ $\triangle ACE \cong \triangle DCE$, so $\angle CEA = \angle CED =$ ∟