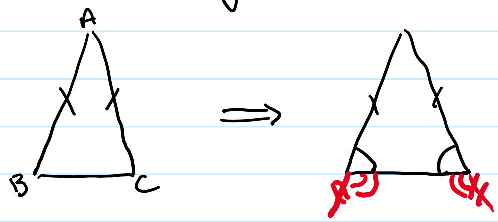
Lecture 3

Thursday, January 11, 2024 8:52 AM

I-5: Isosceles triangles have equal base angles



Proof $|AB| = |AC|$ is given (to show $\angle ABC = \angle ACB$)

→ apply side-angle-side congruence criteria

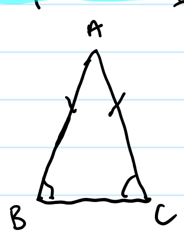
consider $\triangle ABC$ + $\triangle ACB$

we have $|AB| = |AC|$ - given
and $|AC| = |AB|$ - given
and $\angle BAC = \angle CAB$

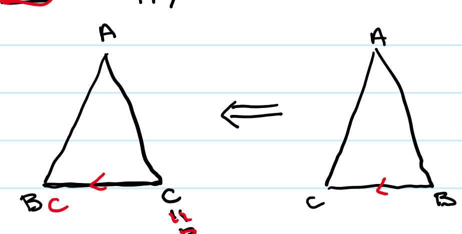
by proposition I-4 - side-angle-side congruence we have
$\triangle ABC \cong \triangle ACB$

$\therefore$ corresponding angles are equal for the 2 triangles
in particular, $\angle ABC = \angle ACB$. //

Proposition I-6: Suppose $\triangle ABC$ has $\angle ABC = \angle ACB$
Then, $|AB| = |AC|$ - prove



Proof: Apply $\triangle ACB$ to $\triangle ABC$. with B being put on C, and BC lying along CB, and A on the same side of CB as A.



(B vertex on C vertex)

since $|BC| = |CB|$ and BC lies along CB and B is on C, we must have C on B.
Still need A on A - How?

(B vertex on C vertex) Still need A on A - How?

Since $\angle ACB = \angle ABC$, CA lies on BA, and since $\angle ABC = \angle ACB$, BA lies on CA

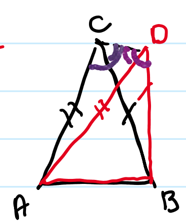
intersect

$\therefore A = CA \cap BA$
$= BA \cap CA$
$= A$

Since A is on A, B is on C and C is on B, we have $|AB| = |AC|$. //

Proposition I-7: Suppose we are given AB and C (which is not any extension of AB). Then any pt D on the same side of AB as C and w $|AD| = |AC|$ and $|BD| = |BC|$, is actually C.

Proof



Suppose by way of contradiction that $C \neq D$

Since $|AC| = |AD|$ and $|BC| = |BD|$, ΔACD and ΔBCD are isosceles. It follows by I-6, that $\angle ACD = \angle ADC$ and $\angle BCD = \angle BDC$.

Then, in the picture, $\angle BCD < \angle ACD$ and $\angle ADC < \angle BDC$

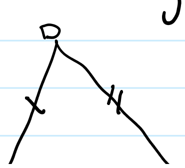
So $\angle BCD < \angle BDC$ and $\angle BCD = \angle BDC$ $\otimes$ contradiction!

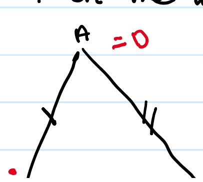
$\therefore D = C$. //

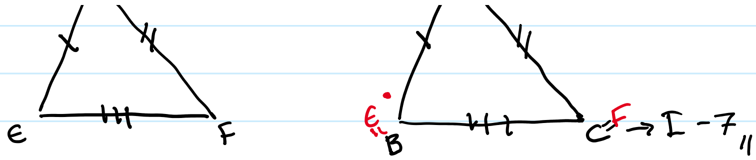
Proposition I-8 - aka side-side-side congruence criterion:

- If ΔABC and ΔDEF have corresponding sides equal. $[|AB| = |DE|, |AC| = |DF|, \text{and } |BC| = |EF|]$ then $\Delta ABC \cong \Delta DEF$.

Proof apply ΔDEF onto ΔABC and use I-7! w D on A, and DE lies along AB, and F on the same side of AB as C.






E
F
E
B
C
F
→ I - 7