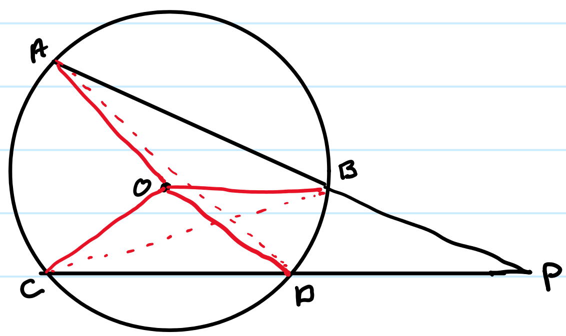
Lecture 17

Friday, February 16, 2024 12:16 PM

Angles and side lengths Propositions Continued



Suppose chords AB and CD of a circle intersect at a pt P outside the circle. Then $|AP| \cdot |BP| = |CP| \cdot |DP|$, and $\angle APC = \frac{1}{2}\angle AOC - \frac{1}{2}\angle BOD$

Proof Angles first...

connect each of A, B, C, D, to the centre O.

$\angle AOC = 2\angle ABC = 2\angle ADC$

$\angle BOD = 2\angle BAD = 2\angle BCD$

connect A to D to make ΔADP

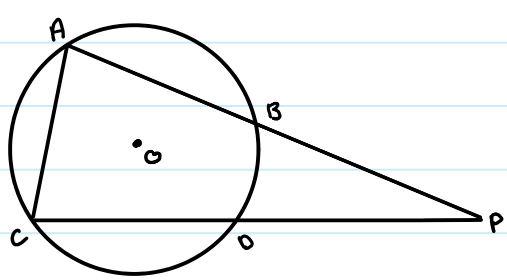
$\angle PAD = \angle BAD = \frac{1}{2}\angle BOD$

$\angle ADC = \frac{1}{2}\angle AOC = \angle PAD + \angle APD$

$= \angle BAD + \angle APC$

$\angle APC = \frac{1}{2}\angle AOC - \frac{1}{2}\angle BOD$ //

*Next...



$\frac{|AP|}{|DP|} = \frac{|CP|}{|BP|}$ would be true if $\Delta APC \sim \Delta DPB$

we have $\angle APC = \angle DPB$ (same angle)

we need either $\angle ACP = \angle DBP$ or $\angle PAC = \angle PDB$

connect B to C to make ΔBCP then $\angle PDB = \angle BCD + \angle CBD$

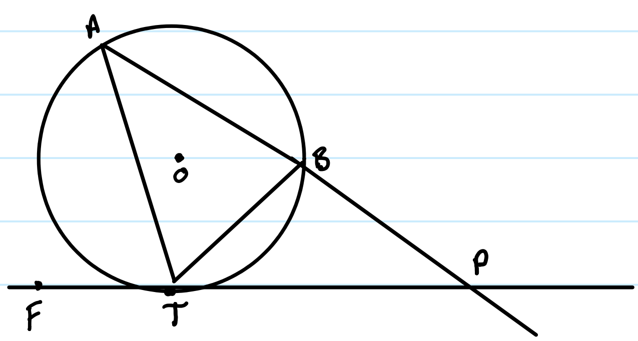
Connect A to D then $\angle BAC = \angle BAD + \angle CAD$

$= \angle BCD + \angle CBC$

$\therefore \Delta APC \sim \Delta DPB$

$\Rightarrow \frac{|AP|}{|DP|} = \frac{|CP|}{|BP|} \Rightarrow |AP| \cdot |BP| = |CP| \cdot |DP|$ //

Proposition: Suppose PT is tangent to a circle a T, and chord AB of the circle meets AB at P.



Then $|AP| \cdot |BP| = |PT|^2$.

Proof connect A to T and B to T, making for three triangles: ΔAPT, ΔTPB, and Triangle ΔABT.

We'll show that $\Delta APT \sim \Delta TPB$: $\angle APT = \angle TPB$ (same angles)

$\angle PBT$ is an external angle of ΔABT

$\parallel$

So $\angle BAT + \angle ATB = \angle PAT + \angle ATB$

$\angle ATF = 2\angle - \angle ATB - \angle BTP$

$\parallel$

$\angle BAT + \angle APT$

* not finished... next time!