

Lecture 16

Friday, February 16, 2024 12:16 PM

The Assignment Plan:

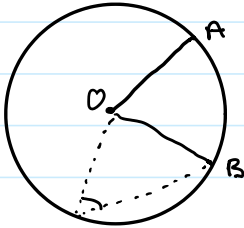
Assignment #5 - due this Friday (take extra time if needed)

Assignment #2e - live on Friday, due Feb, 26th

Assignment #6 - live on Feb 23rd, due March 1st

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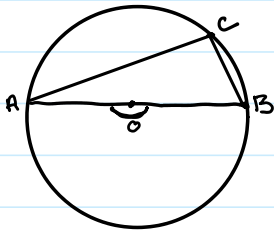
Thales Theorem / Angle/side lengths propositions



A central angle is twice the corresponding circumferential angle. Assignment #5 (III-203)

Corollary: Given a chord, all the "inscribed" angles from the same arc are subtended by the chord are equal.

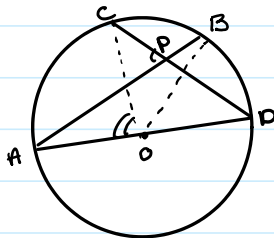
Thales Theorem:



The triangle formed by a diameter and any other point on the circle is right.

Proof $\angle ACB = \frac{1}{2} \angle AOB$
 $= \frac{1}{2} \overset{\frown}{AB} = 90^\circ$

Proposition



* $\angle APC = 2 \angle B = \overset{\frown}{AB}$

Suppose chords AB and CD of a circle intersect at P, then
 $\angle APC = \frac{1}{2} \angle AOC + \frac{1}{2} \angle BOD$

Proof

$\angle APC$?

Proof

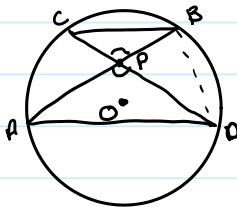
$$\angle APC ?$$

$$\angle ADC = \frac{1}{2} \angle AOC$$

$$\angle BAD = \frac{1}{2} \angle BOD$$

connect A to D to make $\triangle ADP$. Then the external $\angle APC = \angle PAD + \angle PDA$
 $= \angle BAD + \angle ADC$
 $= \frac{1}{2} \angle AOC + \frac{1}{2} \angle BOD //$

Proposition Suppose chords AB and CD meet at P inside the circle
then $|PA| \cdot |PB| = |PC| \cdot |PD|$



Proof connect B to C and A to D to make $\triangle PBC$ and $\triangle PAD$

$$\angle BPC = \angle APD \text{ (opposite angles)}$$

$$\angle ABC = \angle ADC \text{ (Subtend the same chord of AC)}$$

$$|| \quad ||$$

$$\angle PBC = \angle PAD$$

By angle-angle similarity $\triangle PBC \sim \triangle PAD$

$$\Rightarrow \frac{|PB|}{|PD|} = \frac{|PC|}{|PA|} \text{ cross multiply } \Rightarrow |PA| \cdot |PB| = |PC| \cdot |PD| //$$