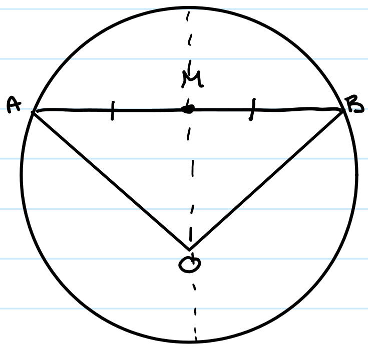
Lecture 15

Wednesday, February 14, 2024 9:37 AM

on to circles (mostly in Book III) and related matters ...

Corollary to III-1: The perpendicular bisector of the chord of a circle passes through the centre of the circle.



Proof: Let M be the midpoint of AB. Draw the line from M to O and also from A to O and B to O.

$|AO| = |BO|$ (radii)

$|MO| = |MO|$

$|MA| = |MB|$ (choice of M)

$\therefore \triangle AOM \cong \triangle BOM$

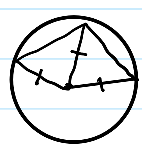
$\Rightarrow \angle AMO = \angle BMO$

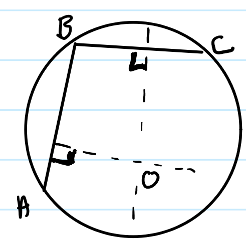
$\Rightarrow \angle AMO + \angle BMO = \angle AMB =$ $\Big\}$ $\Rightarrow \angle AMO = \angle BMO = \llcorner$

III-1 to locate the centre of a given circle

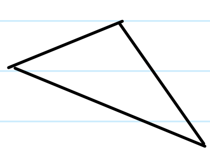
other consequences:

1) Any three non-collinear points in the plane define a unique circle

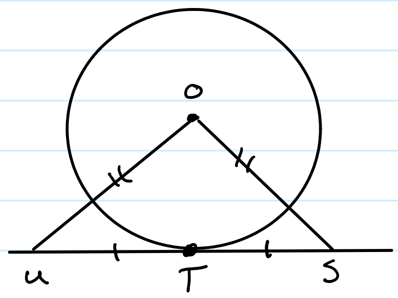




2) Every triangle is circumscribed by a unique circle



Corollary to III-16



If O is the centre of a circle and OT is a radius, then a line passing through T is tangent to the circle if and only if the line is perpendicular to OT.

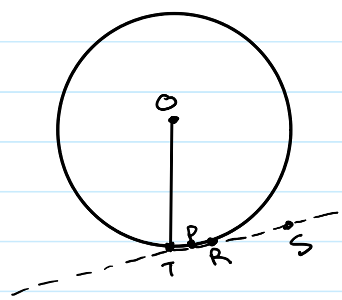
Proof

$\Rightarrow$ Assume that ST is tangent to the circle at T.

To show: OT $\perp$ ST
↳ 'perpendicular to'

Extend ST past T to U so that $|ST| = |UT|$.
connect OU and OT.

Try from other direction
$\Leftarrow$ show the contrapositive ie assume ST is not tangent and show that OT is not perpendicular to ST



if ST is not tangent, then it meets the circle at one or more points R. Draw the perpendicular bisector of TR' at P (the midpt of TR)

$\triangle$ OPT has a right angle, $\angle OPT$ $\therefore$ Since the sum of the internal angles is 2 right angles.

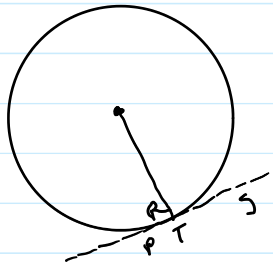
$$\angle OTP = \angle OTS < \llcorner$$
ie) $OT \not\perp ST$

Back to $\Rightarrow$ (assume that ST is tangent and show that $ST \perp OT$)
contrapositive again!

Assume $ST \not\perp OT$, we may assume (by putting S on the otherside of T if

Assume $ST \not\perp OT$, we may assume (by putting S on the otherside of T if necessary)

↳ That $\angle OTS < \llcorner$.



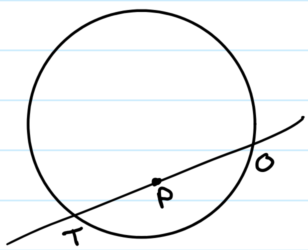
Let OP be the perpendicular to ST from O

(P is on ST) consider $\triangle OPT$

$\angle OPT = \llcorner$

$\Rightarrow \angle OTP < \llcorner$

$\Rightarrow |OT| > |OP|$ $\quad \therefore$ P is inside the circle.



Extend TP past P far enough, and the line will have to meet the circle, so ST is not a tangent. //