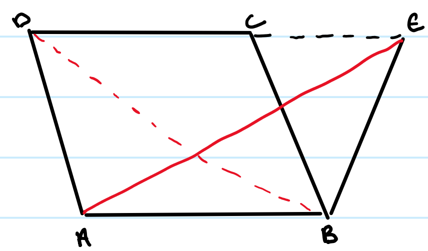
Lecture 13

Friday, February 9, 2024 5:26 PM

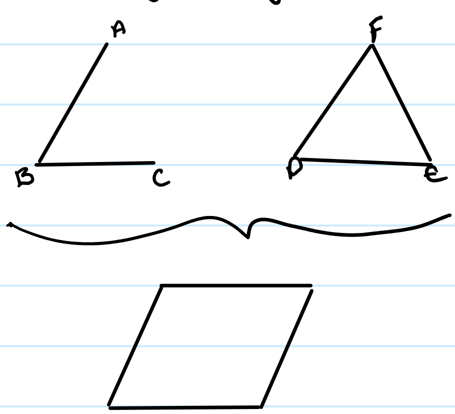
I-41: if a parallelogram has the same base as a triangle and they are between the same parallels, the parallelogram has twice the area of the triangle.



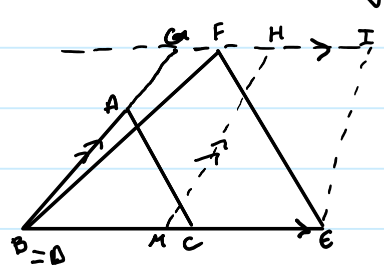
Area(ΔABD) = $\frac{1}{2}$ area ($ABCD$)

= area(ΔABE) (by I - ?)

I-42: To construct a parallelogram equal in area to a given triangle in a given angle



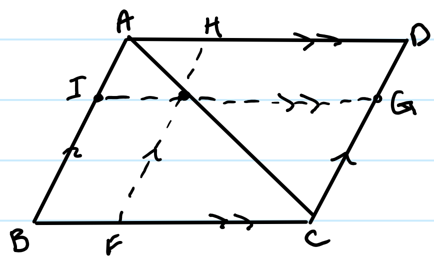
Proof: Connect A to C to make ΔABC. Apply ΔDEF to ΔABC, so that D is on B, DE lies along BC and F is on the same side of BC as A.



construct a line parallel to AB through M, intersecting the line GF at H.

Let M be the midpoint of DE. construct a line through F parallel to DE, which intersects BA at G. This gives a parallelogram BMHG. make a line parallel to BG through E, meeting GF at I; making another parallelogram BEIG. Then by I-41 $\triangle DEF$ has half the area of BEIG but so does DMHG since it has half the base of BEIG

I-43:



Claim: area (EGDH) = area (BFEI)

Proof: $\triangle ABC \cong \triangle CDA$, $\triangle AIE \cong \triangle EHA$, and $\triangle EFC \cong \triangle CGE$

area ($\triangle ABC$) = area ($\triangle CDA$)

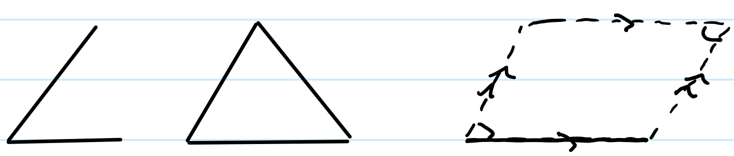
= area ($\triangle AIE$) = area ($\triangle EHA$)

+ area ($\triangle EFC$) = + area ($\triangle CGE$)

+ area (BFEI) = + area (EGDH)

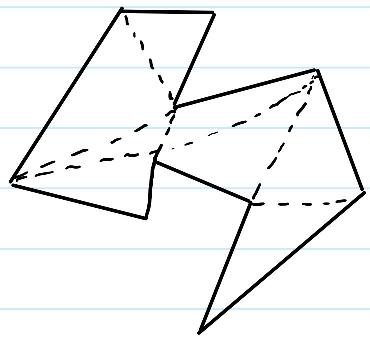
$\therefore$ area (BFEI) = area (EGDH)

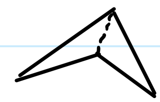
I-44: Given an angle, triangle and a line, construct a parallelogram on that line, using the given angle. that has an area equal to the triangle.



Proof: construct a copy of the angle at a point on the line and then follow the proof of I-42 //

I-45: Given a polygon, construct a parallelogram equal in area to the polygon





Proof: (assuming the arbitrary polygons can be dissected into triangles)
Suppose the polygon has been dissected into triangles: $T_1, T_2, T_3, \ldots, T_x$
construct a parallelogram equal in area to T.