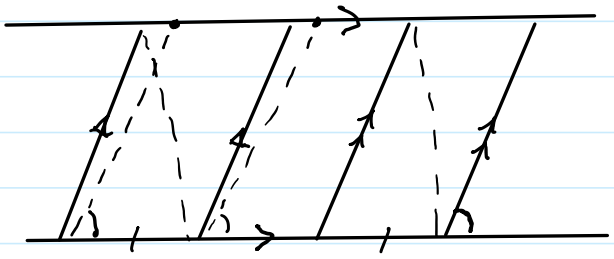
Lecture 12

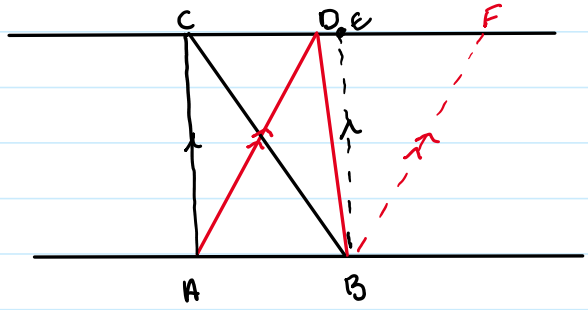
Tuesday, February 6, 2024 8:42 AM

I-36: Parallelograms between the same parallels and on equal bases have the same area.



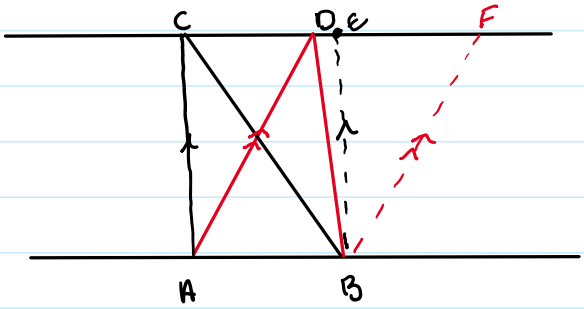
Back to I-35.

I-37: Triangles with same bases and between the same parallels have equal bases.



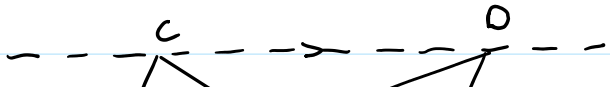
Proof area (ABC)
$= \frac{1}{2}$ area (ABEC)
$= \frac{1}{2}$ area (ABFD)
= area (ABD

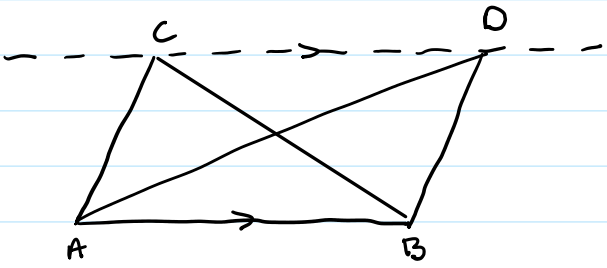
I-38: Triangles with equal bases and between the same parallels have equal bases.



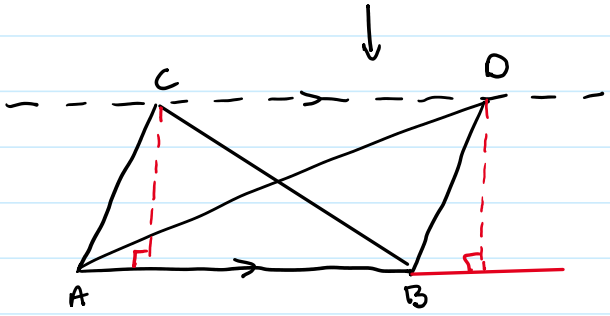
Proof area (ABC)
$= \frac{1}{2}$ area (ABEC)
$= \frac{1}{2}$ area (ABFD)
= area (ABD

I-39: Triangles with the same bases and with equal area are between the same parallels [ie have same height]

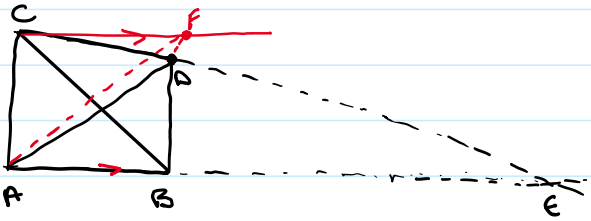




Given ΔABC and ΔABD with equal areas, $AB \parallel CD$.



Assume by way of contradiction ⊗ that CD is not parallel to AB.



Draw a parallel to AB through C and let F be the the pt where BD meets the new line.

Connect A to F to make ΔABF, by I-37 ΔABF and ΔABC have equal areas.

1) if D is between B and F, ΔABD is part of ΔABF.
2) if F is between B and D, then ΔABF is part of ΔABD.

In either case ΔABD and ΔABF have different areas, and hence ΔABD has a different area from ΔABC contradicting the assumption they are equal.

$\therefore$ CD must be $\parallel$ (parallel) to AB.

I-40: Triangles on equal bases, (on same line) and equal areas are between the same parallels.

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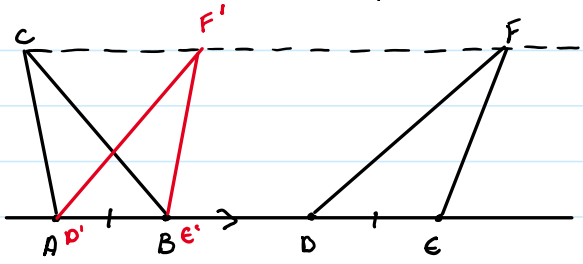
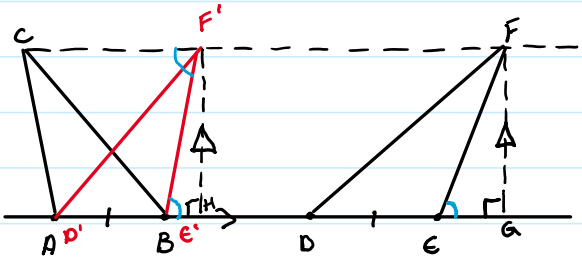


Proof: Apply ΔDEF to ΔABC so that $BC = D'E'$ and then $CF' \| AB \| D'E$ by I-39. Why is F on CF' too?



By A-A-S congruence ($\cong$)

$\Delta BHF' \cong \Delta EGF$

so $|HF'| = |GF|$.

by I-33 we then have a parallelogram.