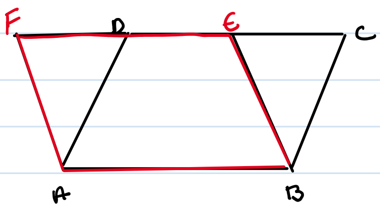
Lecture 11

Thursday, February 1, 2024 11:53 AM

Areas Euclid's way (ie. via parallelograms)

I-35: Two parallelograms → ▱ w the same base and between the same parallels have equal areas.

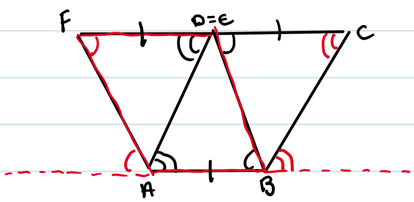
Proof:



we have ▱ ABCD and ABEF w CDEF on the same line.

* we'll work by cases based on how the ▱'s overlap.

case 0: D = E

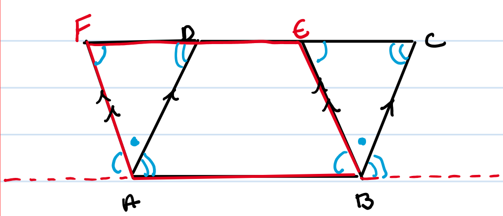


$|AB| = |FD| = |CD|$

By S-A-S we can deduce that $\Delta DAB \cong \Delta BCD$
$\cong \Delta ADF$

$$\begin{aligned} \text{area}(ABCD) &= \text{area}(\Delta ABD + \Delta DCB) \\ &= \text{area}(\Delta ABD + \Delta ADF) \\ &= \text{area}(ABEF) \end{aligned}$$ //

case 1:



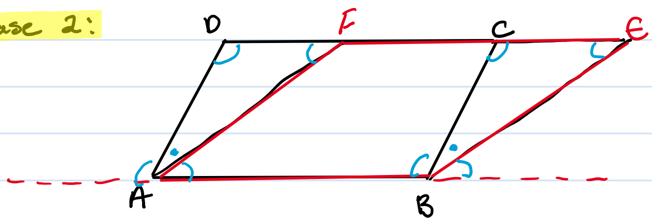
we must have $|AF| = |BE|$ and $|AD| = |BC|$.

$\angle = \angle$, $\angle = \angle$, and $|AD| = |BC|$

$\Rightarrow \Delta BCE \cong \Delta ADF$

$$\Rightarrow \text{area}(ABCD) = \text{area}(ABED) + \text{area}(\Delta BCE)$$
$$= \text{area}(ABED) + \text{area}(\Delta ADF)$$
$$= \text{area}(ABEF)$$ //.

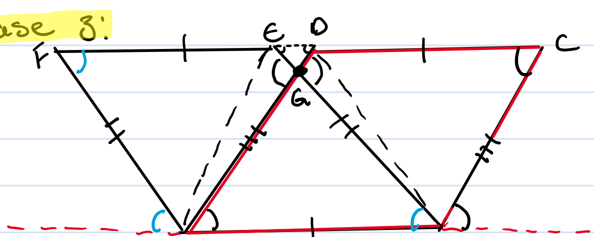
Case 2:



S-A-S (using $|AF| = |BE|$ and $\angle = \angle$ and $\angle = \angle$).

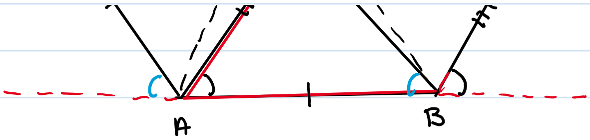
* And decomposes similarly to case (1).

case 3:



By S-A-S, $\Delta ADF \cong \Delta BCE$.

$$▱ ABEF = |\Delta ADF| + |\Delta ABG| - |\Delta GED|$$
$$= |\Delta BCE| + |\Delta ABG| - |\Delta GED|$$
$$= |▱ ABCD|$$ //



$= |\Delta BCE| + |\Delta ABG| - |\Delta GED|$

$= |\text{▱} ABCD|$ //.