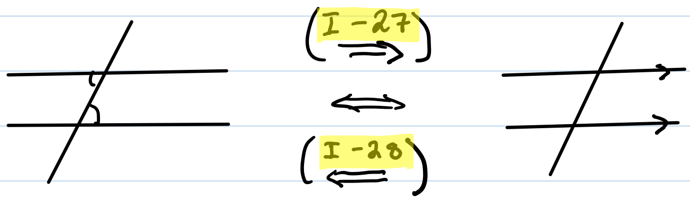
Lecture 10

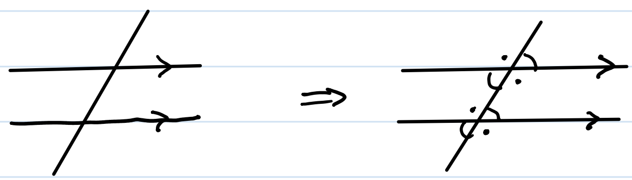
Thursday, February 1, 2024 8:52 AM

Today we explore parallels . . .

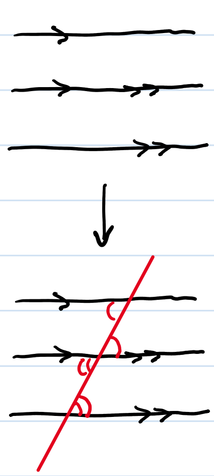
Recall: prop I-27 and I-28 (Z - theorem)



I-29:

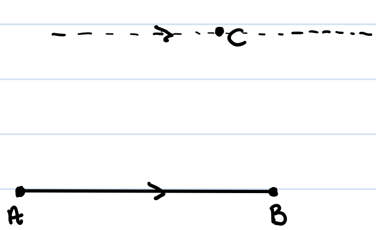


I - 30: If two lines are each parallel to a third, they are also parallel to each other

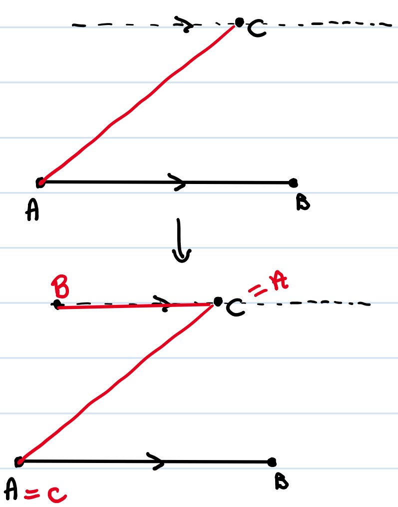


$\rightarrow \measuredangle = \measuredangle$

I - 31: Given a line AB and a point C, not on any extension of AB, we can draw a line through C parallel to AB.



Proof: Draw a line from A to C.

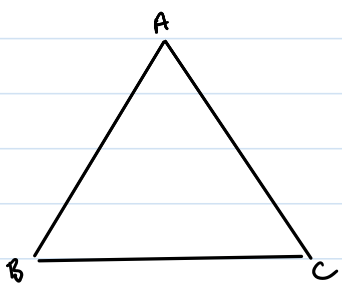


Consider $\angle CAB$. Draw a copy of $\angle CAB$ at C so that AC falls along CA and the rest of the new angle is on the other side of CA from B.

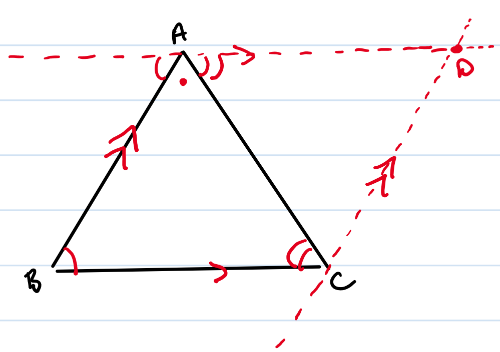
This makes alternate interior angles, so AB $\parallel$ AB. //

"parallel to"

$\boxed{\text{I-32:}}$ The sum of the interior angles of any triangle is equal to the sum of two right angles.



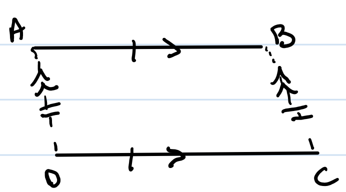
Proof:



Draw a line through A parallel to BC. Then $\angle EAB = \angle ABC$, and $\angle DAC = \angle ACB$ by Z-theorem.

$$\angle ABC + \angle ACB + \angle BAC$$
$$= \angle EAB + \angle DAC + \angle BAC$$
$$= \angle EAD = 2L. //$$

$\boxed{\text{I-33:}}$

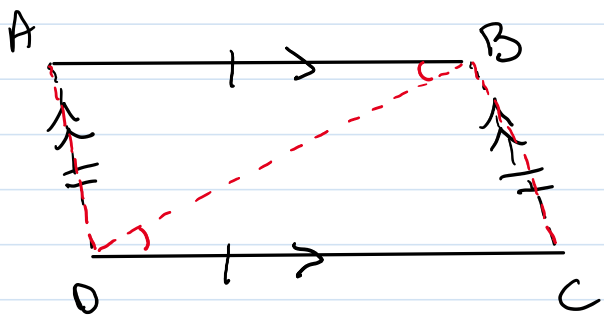


Suppose AB $\parallel$ DC and $|AB| = |DC|$, Then AD $\parallel$ BC and $|AD| = |BC|$.

Proof:

Draw BD, then by the Z-thm

Proof:



Draw BD, then by the Z-thm $\angle ABD = \angle CDB$

Since $|AB| = |DC|$ and $|BD| = |DB|$, S-A-S gives us $\triangle ABD \cong \triangle CDB$

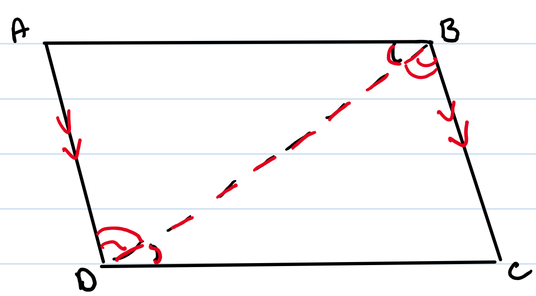
Thus, $|AD| = |CB|$, and $\angle CBD = \angle ADB$

$\Downarrow$ by Z-thm

$AD \parallel CB$. //

I-34: In any parallelogram, opposite sides are equal in length, and opposite angles are equal, and a diagonal "cuts it in half".

Proof: Given ▱ABCD.



Draw the diagonal BD. I-1

By Z-thm we have $\angle ABD = \angle CDB$ and also $\angle ADB = \angle CBD$

Thus $\angle ABC = \angle ABD + \angle CBD$
$= \angle CDB + \angle ADB$
$= \angle ADC$.

and $\angle BAD = 2\llcorner - \angle ABD - \angle ADB$
$= 2\llcorner - \angle CDB - \angle CBD$
$= \angle BCD$.

~ because

$= \angle BCD$.

By A-S-A congruence ($\cong$) we have $\Delta ABD \cong \Delta CDB$ b/c (because) $\angle ABD = \angle CDB$ and $\angle ADB = \angle CBD$ and $|BD| = |DB|$, $\therefore$ means $|AD| = |CB|$ and $|AB| = |CD|$. //