Lecture 32: Tiling the Whole plane with Kites, Darts & Logic

April 4, 2024

We can tile the whole plane with Kites and darts and Logic'

If you have a collection of statements in a propositional or first order language, then the collection is Consistent if you can't deduce a contradiction from it

A collection or set of statements is not Consistent ("inconsistent") if you derive a contradiction eg $\neg(\alpha \to \alpha)$ or $\alpha \wedge (\neg\alpha)$

But any such proof is finite & so uses only a finite subset of the whole collection which is inconsistent

A set of statements is consistent iff every finite subset is consistent

~~Sed~~ Soundness Theorem: Every proof preserves truth.

Completeness Theorem If some collection of statements is consistent then some mathematical structure satisfies all of those statements

Soundness Theorem (2.0) Every collection of statements that has a model is consistent.

Compactness Theorem: A set of statements has a structure satisfying it iff every finite subset of it does.

Theorem: we can tile the plane using Penrose's Kites & darts

Proof:

1) Look at the finite part of the "sun" arrangement we had last time

2) By using deflation & rescaling we can get arbitrarily large

- In terms of area covered & the number of tiles used - finite tilings with kites & darts

3) So we can always satisfy a finite collection of statements

That say "there are $\geq n$ in tiles covering an area of $\geq \varphi n$"
↳ golden ratio

Applying the compactness theorem, we can satisfy all of these (ie for all n) at once. So the structure along this is a tiling of the entire plane. //