

April 2, 2024

## Penrose's Kites & Darts tilings (with a side of reptiles)

- Mostly drawn from a column of Martin Gardner's in Scientific American in 1977  
(Expanded in Penrose Tiles to Trapdoor Ciphers Published by the Mathematical Association of America, 1977)

inflation: grow, make larger

deflation: Shrink, make smaller

### Kite & Dart tile specifications:

- Start with a finite Kites & Darts pattern
- Apply deflation to chop into smaller Kites and darts
- rescale to make the small ones be the original size
- Repeat...

⑤ - not identical

Aside: A reptile is a shape that can be dissected into congruent (to each other) smaller copies of the shape



### 5-fold symmetry:

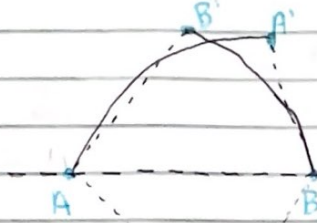
- A point around which rotations of multiples of  $72^\circ$  don't change the tiling
- A point around which there are five lines each of which has reflection symmetry for the tiling

**Theorem (Barlow):** Any pattern in the plane with five-fold symmetry has an unique centre for that symmetry.

(proof on following page)

Proof:

Suppose  $A$  is a point of symmetry for some pattern.



Assume by way of contradiction, that there is at least one other point of five-fold symmetry for the pattern.

Let  $B$  be one of these other points of symmetry that is as close to  $A$  as possible

(In general, pick two symmetry points that are as close as possible)  $\rightarrow$  counterclockwise

Rotate  $B$   $72^\circ$  about  $A$  to get  $B'$ , & rotate  $A$   $72^\circ$  clockwise about  $B$  to get  $A'$

This puts  $A', B'$  closer together than  $A, B$   
 $\otimes$  to  $(AB)$  being the minimum distance...

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