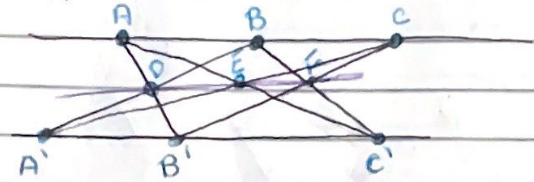
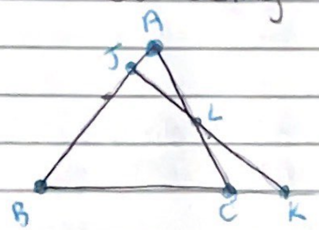
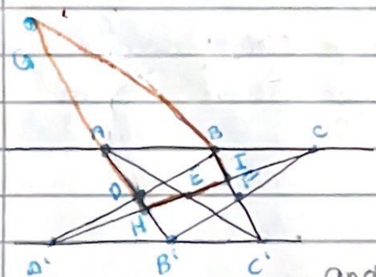
Pappus' Theorem



We'll be using Menelaus' theorem (a lot)



J, K, L are collinear if: $\frac{AJ}{JB} \cdot \frac{BK}{KC} \cdot \frac{CL}{LA} = -1$



Extend B'A and C'B to intersect at G and let AB' & A'C intersect at H, and let BC' & A'C intersect at I. Consider $\triangle GHI$.

DEF collinear if: $\frac{GD}{DH} \cdot \frac{HE}{EI} \cdot \frac{IF}{FG} = -1$

Consider the following "transversals" of $\triangle GHI$:

① $AEC' \Rightarrow \frac{GA}{AH} \cdot \frac{HE}{EI} \cdot \frac{IC'}{C'G} = -1$

② $A'DB \Rightarrow \frac{GD}{DH} \cdot \frac{HA'}{A'I} \cdot \frac{IB}{BG} = -1$

③ $B'FC \Rightarrow \frac{GB'}{B'H} \cdot \frac{HC}{CI} \cdot \frac{IF}{FG} = -1$

④ $ABC \Rightarrow \frac{GA}{AH} \cdot \frac{HC}{CI} \cdot \frac{IB}{BG} = -1$

⑤ $A'B'C' \Rightarrow \frac{GB'}{B'H} \cdot \frac{HA'}{A'I} \cdot \frac{IC'}{C'G} = -1$

①·②·③: $= (-1)^3 = -1$

$$\frac{GA}{AH} \cdot \frac{HE}{EI} \cdot \frac{IC'}{C'G} \cdot \frac{GD}{DH} \cdot \frac{HA'}{A'I} \cdot \frac{IB}{BG} \cdot \frac{GB'}{B'H} \cdot \frac{HC}{CI} \cdot \frac{IF}{FG}$$

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$$\frac{\textcircled{1}\cdot\textcircled{2}\cdot\textcircled{3}}{\textcircled{4}}:$$

$$\frac{\cancel{\frac{GA}{AH}}\cdot\frac{HE}{EI}\cdot\cancel{\frac{IC'}{C'G}}\cdot\frac{GD}{DH}\cdot\cancel{\frac{HA'}{A'I}}\cdot\cancel{\frac{IB}{BG}}\cdot\cancel{\frac{GB'}{B'H}}\cdot\cancel{\frac{HC'}{C'I}}\cdot\frac{IF}{FG}}{\cancel{\frac{GA}{AH}}\cdot\cancel{\frac{HC'}{C'I}}\cdot\cancel{\frac{IB}{BG}}\cdot\cancel{\frac{GB'}{B'H}}\cdot\cancel{\frac{HA'}{A'I}}\cdot\cancel{\frac{IC'}{C'G}}}$$

$$= \frac{HE}{EI}\cdot\frac{GD}{DH}\cdot\frac{IF}{FG} = -1$$

Desargue's theorem:

Two triangles are in perspective from a point iff they are in perspective from a line