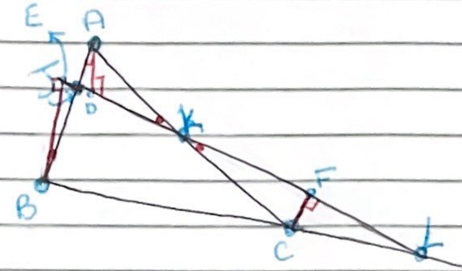


Suppose J, K, & L are points on (extensions of) the sides of $\triangle ABC$, namely AB, AC, & BC, resp. Then J, K, L are collinear if and only if

$$\frac{AJ}{JB} \cdot \frac{BL}{LC} \cdot \frac{CK}{KA} = 1$$

Proof: Suppose J, K, L are collinear. ($\boxed{\Rightarrow}$)
Draw perpendiculars to JK from A, B & C meeting the line JK at D,E,F resp.

$\angle ADK = \angle CFK = ⊾$ &
$\angle AKD = \angle CKF$ (Opposite angles)

$\therefore \triangle ADK \sim \triangle CFK \Rightarrow \frac{|AK|}{|KC|} = \frac{|FK|}{|FD|} = \frac{|AD|}{|CF|}$

Similarly, $\triangle BLE \sim \triangle CLF$

$$\Rightarrow \frac{|CL|}{|BL|} = \frac{|LF|}{|LE|} = \frac{|CF|}{|BE|}$$

Similarly, $\triangle ADJ \sim \triangle BEJ$

$$\Rightarrow \frac{|BE|}{|AD|} = \frac{|BJ|}{|AJ|} = \frac{|EJ|}{|DJ|}$$

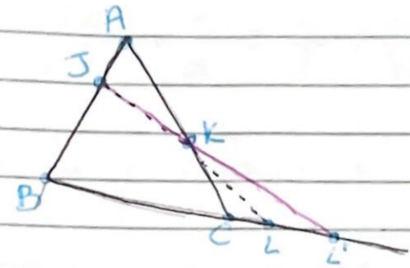
$$\underset{+}{\frac{AJ}{JB}} \cdot \underset{-}{\frac{BL}{LC}} \cdot \underset{+}{\frac{CK}{KA}} = 1 = -\frac{|AJ|}{|JB|} \cdot \frac{|BL|}{|LC|} \cdot \frac{|CK|}{|KA|}$$

$$= -\frac{\cancel{|AD|}}{\cancel{|BE|}} \cdot \frac{\cancel{|BE|}}{\cancel{|CF|}} \cdot \frac{\cancel{|CF|}}{\cancel{|AD|}} = -1 \text{ (as desired)}$$

$\boxed{\Leftarrow}$ Assume $\frac{AJ}{JB} \cdot \frac{BL}{LC} \cdot \frac{CK}{KA} = 1$

Join J to K & extend to L' on BC.
(as seen in diagram on next page)

March 14, 2024



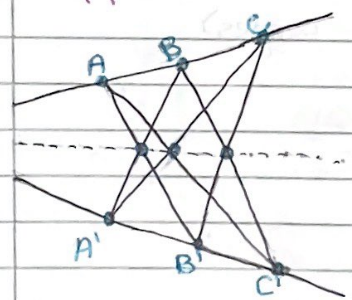
By $\boxed{\Rightarrow}$,

$$\frac{AJ}{JB} \cdot \frac{BL'}{L'C} \cdot \frac{CK}{KA} = -1 = \frac{AJ}{JB} \cdot \frac{BL}{LC} \cdot \frac{CK}{KA}$$

$$\Rightarrow \frac{BL'}{L'C} = \frac{BL}{LC} \Longrightarrow L = L'$$

& so J, K, L are collinear //

Pappus' Theorem



Pascal