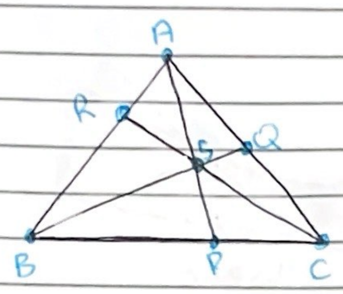
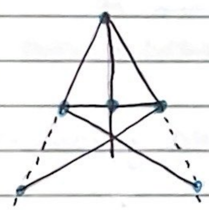


The cevians AP, BQ and CR are concurrent if and only if:

$$\frac{AR}{RB} \cdot \frac{BP}{PC} \cdot \frac{CQ}{QA} = 1$$

Proof: Lots of areas of triangles, so we'll write the area of $\triangle XYZ$ as $|\triangle XYZ|$ for convenience

$\Rightarrow$ Assume the cevians are concurrent in S.

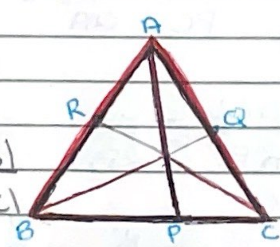


Observe that $\triangle APB$ and $\triangle APC$ have a common height h, and then $|\triangle APB| = \frac{1}{2} \cdot h \cdot |BP|$ & $|\triangle APC| = \frac{1}{2} \cdot h \cdot |PC|$.

$$\therefore \frac{BP}{PC} = \frac{|\triangle APB|}{|\triangle APC|}$$

Similarly, $\triangle SPB$ & $\triangle SPC$ have areas proportional to $|BP|$ & $|PC|$

ie $\frac{BP}{PC} = \frac{|\triangle SPB|}{|\triangle SPC|} = \frac{|\triangle APB|}{|\triangle APC|}$



Claim: $\frac{BP}{PC} = \frac{|\triangle ASB|}{|\triangle ASC|} = \frac{|\triangle APB| - |\triangle SPB|}{|\triangle APC| - |\triangle SPC|}$

Lemma:

$$\frac{a}{b} = \frac{c}{d}$$

$$\Rightarrow \frac{a-c}{b-d} = \frac{a}{b} = \frac{c}{d}$$

Proof: Suppose $k = a/c$ then $a = kc$ and we must have $b = kd$. Then $\frac{a-c}{b-d} = \frac{kc-c}{kd-d}$

$$= \frac{\cancel{(k-1)}}{\cancel{(k-1)}} \cdot \frac{c}{d} = \frac{c}{d} = \frac{a}{b} \;//$$

Claim is therefore true by Lemma.

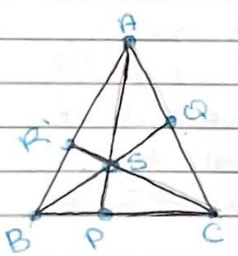
Similarly, $\frac{AR}{RB} = \frac{|\Delta ASC|}{|\Delta BSC|}$ & $\frac{CQ}{QA} = \frac{|\Delta BSC|}{|\Delta ASB|}$

Thus $\frac{AR}{RB} \cdot \frac{BP}{PC} \cdot \frac{CQ}{QA} = \frac{\cancel{|\Delta ASC|}}{\cancel{|\Delta BSC|}} \cdot \frac{\cancel{|\Delta ASB|}}{\cancel{|\Delta ASC|}} \cdot \frac{\cancel{|\Delta BSC|}}{\cancel{|\Delta ASB|}} = 1$

as required

$\Leftarrow$ Suppose $\frac{AR}{RB} \cdot \frac{BP}{PC} \cdot \frac{CQ}{QA} = 1$

We need to show that AP, BQ, CR are concurrent

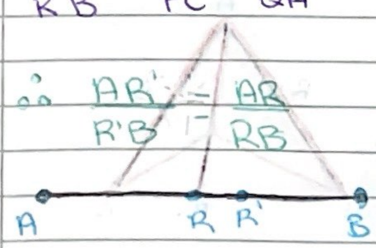


Let S be the intersection of AP & BQ. Draw CS and extend it to R' on AB

By the $\boxed{\Rightarrow}$ we know that:

$$\frac{AR'}{R'B} \cdot \frac{BP}{PC} \cdot \frac{CQ}{QA} = 1 = \frac{AR}{RB} \cdot \frac{BP}{PC} \cdot \frac{CQ}{QA}$$

$\therefore \frac{AR'}{R'B} = \frac{AR}{RB}$



$$\frac{r'}{1-r'} = \frac{r}{1-r} = t$$

$\Rightarrow r = (1-r)t$

$\Rightarrow r = t - rt$

$\Rightarrow r(1-t) = t$

$\Rightarrow r = t/_{1+t}$

$\Rightarrow r = r'$

$\therefore R = R'$ $\therefore$ AP, BQ, CR are concurrent //