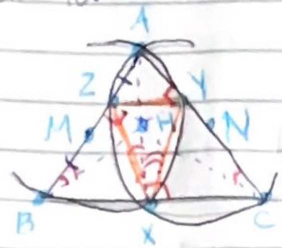
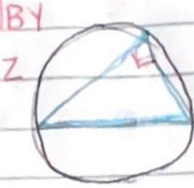
Back to:



$\angle AXY = \angle ABY$

$\angle AXZ = \angle ACZ$



Let X, Y, Z be the feet of the altitudes from A, B, C resp of $\triangle ABC$.
Then AX, BY, CZ bisect the internal angles of $\triangle XYZ$.

Proof: Let M be the midpoint of AB. Draw the circle of radius MA about A. This makes a diameter of the circle.

Claim: X & Y are on the circle. By the converse of Thales' theorem, since $\angle AYB = ⦜ = \angle AXB$, it follows that X & Y are on the circle.

Consider the chord AY of the circle. Since $\angle AXY$ and $\angle ABY$ subtend this chord, they are equal.

Let N be the midpoint of AC. Draw the circle with centre N & radius NA, making AC a diameter. By the converse of Thales' theorem Z & X are on the circle.

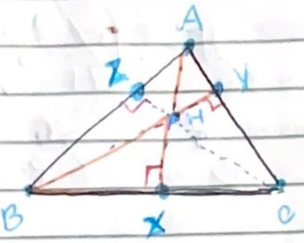
We will show that $\angle ABY = \angle ACZ$.
observe that $\angle BZH = ⦜ = \angle CYH$
and $\angle BHZ = \angle CHY$ (opposite angles)

Thus $\triangle BHZ \sim \triangle CHY$, & so $\underset{\angle AXY}{\underset{\shortparallel}{\angle ABY}} = \angle ZBH = \angle YCH = \underset{\angle AXZ}{\underset{\shortparallel}{\angle ACZ}}$.

$\therefore$ AX bisects $\angle ZXY$. Similarly, BY & CZ bisect $\angle XYZ$ & $\angle XZY$. //

Theorem: Suppose X, Y, Z are the feet of the altitudes from A, B, C resp. in ΔABC & let H be the orthocentre of ΔABC

Then: $|AH| \cdot |HX|$
$|BH| \cdot |HY|$
$|CH| \cdot |HZ|$



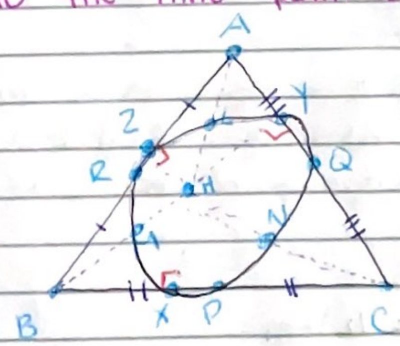
Proof: $\frac{|AH|}{|BH|} = \frac{|HY|}{|HX|}$? $\Delta ABH \sim \Delta YXH$?

Consider ΔAHY & BHX.
$\angle AYH = \llcorner = \angle BXH$
$\angle AHY = \angle BHX$ (opposite angles)
$\Rightarrow \Delta AHY \sim \Delta BHX$

$\Rightarrow \frac{|AH|}{|HY|} = \frac{|BH|}{|HX|} \rightarrow |AH| \cdot |HX| = |BH| \cdot |HY|$

Similarly, we can show that $|BH| \cdot |HY| = |CH| \cdot |HZ|$ //

Intro to the nine point circle:



Let P, Q, R be the midpoints of sides BC, AC & AB resp. Let X, Y, Z be the feet of the altitudes from A, B, C resp.
Let L, M, N be the midpoints of AH, BH & CH resp.

March 7, 2024