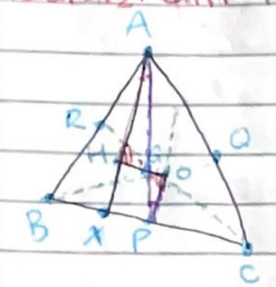
The centroid G, the circumcentre O, and the (Euler) orthocentre H of ΔABC are collinear, and the line G is between O & H with $|OG| = \frac{1}{2}|GH|$ is the Euler line.



Let P, Q, R be the midpoints of BC, AC, AB resp.

Extend OG past G to H such that $|GH| = 2|GO|$

Claim: H is the orthocentre of ΔABC, ie each altitude of ΔABC

Connect A to H and extend the line past H until it meets BC at x. We'll show that Ax is an altitude, ie that Ax is perpendicular to BC.

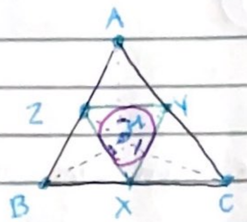
$\angle AGH = \angle PGO$ (opposite angles) &
$|GH| = 2|GO|$ (by construction)
$|AG| = 2|PG|$ since the centroid divides each median in a 2:1 ratio
By S-A-S Similarity $\Delta AGH \sim \Delta PGO$ (in a 2:1 ratio)

it follows that by the similarity $\angle OPG = \angle HAG$
By the Z-theorem, since AP falls across Ax and OP, we have $Ax \parallel OP$
It now follows that Ax & OP meet BC at the same angle, ie $\angle AxP = \angle OCP = \llcorner$

$\therefore$ Ax is an altitude, passes through H.

Similar arguments can be G used to show that the altitudes from B and C also pass through H, so it is the orthocentre

Theorem. Suppose H is the orthocentre of ΔABC & X, Y, Z are the "feet" of the altitudes from A, B, C resp.
(ie AX is the altitude from A, ...)



Then H is the incentre of ΔXYZ and $|AH| \cdot |HX| = |BH| \cdot |HY| = |CH| \cdot |HZ|$

We'll try to show H is the incentre of ΔXYZ
ie AX, BY, CZ are angle bisectors of ΔXYZ
ie H is equidistant from XY, YZ, ZX.