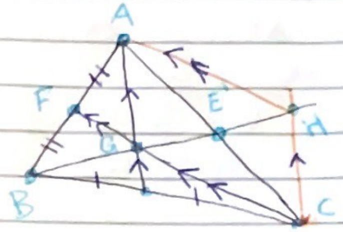
Feb. 29, 2024

Lecture 20: previous question continued



CH∥DG (& CH∥GA...) $\triangle BGD \sim \triangle BHC$ with proportions 1 to 2.

$|AF| = |FB|$ & $|BD| = |DC|$

F is the midpoint of AB

Claim: G is the midpoint of BH

Since $|BG| = \frac{1}{2}|BH|$ because $\triangle BGD \sim \triangle BHC$ with proportions 1 to 2

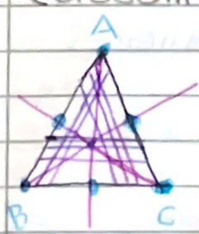
$\therefore$ by one of the lemmas GF∥HA & so GC∥AH

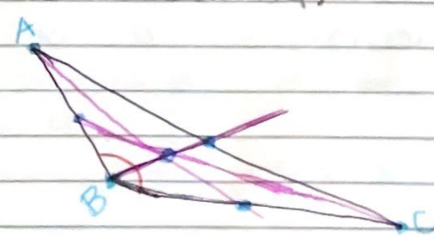
$\therefore$ AGCH is a parallelogram

By the other Lemma, AC & GH bisect each other so E' is the midpoint AC and hence BE is a median & it also passes through G.

We also have that $|BG| = |GH| = 2|GE'|$ & similarly for the other medians

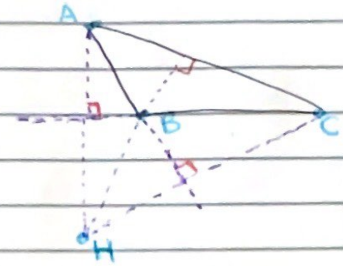
Q: Why is the centroid the centre of mass? (assuming uniform density)



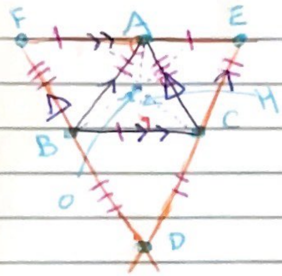


Works with not only equilateral triangles.

Orthocentre: The point where the altitudes of a triangle meet.



Proposition: The altitudes of a triangle are concurrent



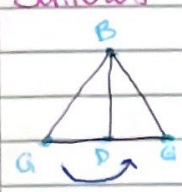
Proof.

Draw lines through each vertex parallel to the opposite side, creating a tangent triangle $\triangle DEF$ then $\triangle ABC \sim \triangle DEF$ and the small ones are all $\cong$

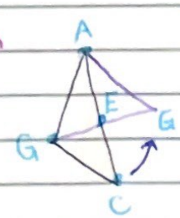
Thus, A, B, C are the midpoints of EF, FD, & DE, resp.

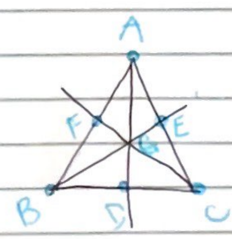
Since A is the midpoint of EF and $BC \parallel EF$, the altitude of $\triangle ABC$ from A is the perpendicular bisector of side EF of $\triangle DEF$. Similarly for the altitudes from B & C. The perpendicular bisectors of the sides of $\triangle DEF$ (ie altitude of $\triangle ABC$) meet in the Circumcentre of $\triangle DEF$...

Sallow's Theorem

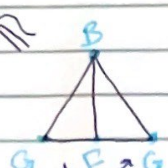


$\cong$





$\cong$



Feb. 29, 2024