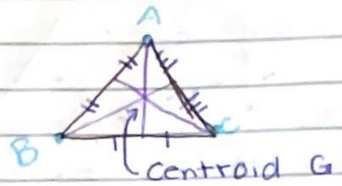
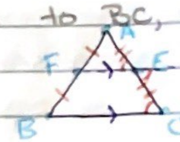
The centroid of a triangle...



Lemma 1: The midpoints E and F of sides AC and AA, respectively, of ΔABC, make a line segment parallel to BC, and half the length of BC.



Proof

$\Delta AFE \sim \Delta ABC$ by S-A-S~ because $\angle AFE = \angle BAC$ (same angle)

$|AF| = \frac{1}{2}|AB|$ & $|AE| = \frac{1}{2}|AC|$

(F and E are midpoints)

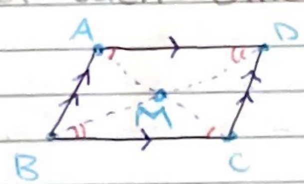
$\therefore |FE| = \frac{1}{2}|BC|$

By similarity, we also get $\angle AEF = \angle ACB$

By the Z-theorem (& it's extentions)

it follows that $FE \parallel BC$ //

Lemma 2: The diagonals of a parallelogram bisect each other



Proof

Let M be the intersection of the diagonals AC and BD

By the Z-theorem, $\angle CAD = \angle BCA$ &

$\angle ADB = \angle CBD$.

By A-A~, it follows that $\Delta AMD \sim \Delta CMB$

Since $|AD| = |BC|$ (opposite sides of parallelogram)

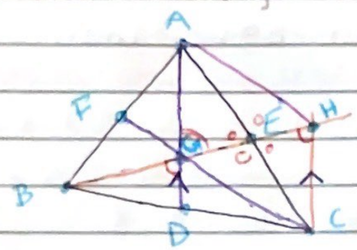
we have $\Delta AMD \cong \Delta CMB$

$\therefore |CM| = |AM|$ & $|BM| = |DM|$, so M is the midpoint of both diagonals //

Lecture 19: Theorem

Theorem: Given any triangle $\triangle ABC$, let D, E & F be the midpoints of sides BC, AC, & AB respectively then the medians AD, BE and CF are concurrent in a single point G, the centroid of $\triangle ABC$.

Also, $|AG| = 2|GD|$, $|BG| = 2|GE|$ & $|CG| = 2|\quad|$



Proof:

Let G be the intersection of AD & CF. Connect B to G and extend it past AC (as necessary) intersecting AC at E'

Draw a line through C parallel to AD and intersecting BG at H

Connect A to H

Since $GD \parallel HC$, $\angle BGD = \angle BHC$ (Z-theorem & extentions),

and $\angle GBD = \angle HBC$ (same angle), so $\triangle GBD \sim \triangle HBC$. So since $|BC| = 2|BD|$, it follows that $|HC| = 2|GD|$ and $|BH| = 2|BG|$

Can we show that AGCH is a parallelog--ram?

In particular, how do we get $AH \parallel GC$?

Feb. 29, 2024