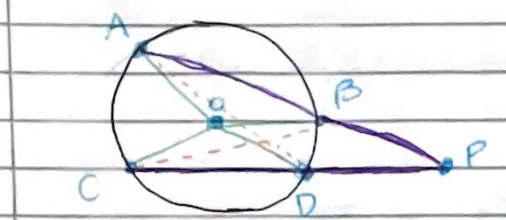
Proposition:



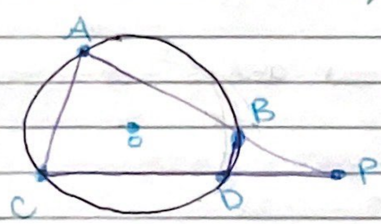
Suppose chords AB & CD of a circle intersect at a point P outside the circle. Then $|AP| \cdot |BP| = |CP| \cdot |DP|$, and
$\angle APC = \frac{1}{2}\angle AOC - \frac{1}{2}\angle BOD$

Proof: Angles first...
Connect each of A, B, C, D to the centre O.

$$\angle AOC = 2\angle ABC = 2\angle ADC$$
$$\angle BOD = 2\angle BAD = 2\angle BCD$$

Connect A to D to make $\triangle ADP$

$\angle PAD = \angle BAD = \frac{1}{2}\angle BOD$
$\angle ADC = \frac{1}{2}\angle AOC = \angle PAD + \angle APD$
$\qquad = \angle BAD + \angle APC$
$\angle APC = \frac{1}{2}\angle AOC - \frac{1}{2}\angle BOD$ //



$\dfrac{|AP|}{|DP|} = \dfrac{|CP|}{|BP|}$ would be true if $\triangle APC \sim \triangle DPB$

We have $\angle APC = \angle DPB$ (same angle)
We need either $\angle ACP = \angle DBP$
or $\angle PAC = \angle PDB$
Connect B to C to make $\triangle BCP$ then
$\angle PDB = \angle BCD + \angle CBD$
Connect A to D. Then $\angle BAC = \angle BAD + \angle CAD$
$= \angle BCD + \angle CBD$

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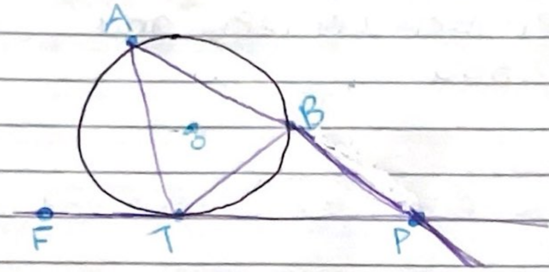
$\therefore \triangle APC \sim \triangle DPB$

$\Rightarrow \dfrac{|AP|}{|DP|} = \dfrac{|CP|}{|BP|} \Rightarrow |AP| \cdot |BP| = |CP| \cdot |DP|$ //

Proposition: Suppose PT is tangent to a circle at T, and chord AB of the circle meets AB at P.



Then $|AP| \cdot |BP| = |PT|^2$

Proof

Connect A to T and B to T, making for the three triangles: $\triangle APT$, $\triangle TPB$, $\triangle ABT$.

We'll show that $\triangle APT \sim \triangle TPB$:

$\angle APT = \angle TPB$ (same angle)

$\angle PBT$ is an external angle of $\triangle ABT$

$\parallel$

So $\angle BAT + \angle ATB = \angle PAT + \angle ATB$

$\angle ATF = 2\llcorner - \angle ATB - \angle BTP$

$\parallel$

$\angle BAT + \angle APT$

* Unfinished - finish next class after reading week