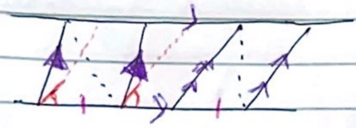
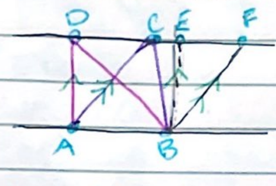
I-36: Parallelograms between the same parallels and on equal bases have the same area.



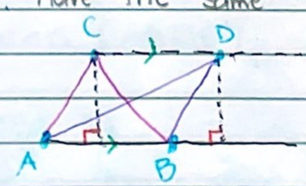
I-37: Triangles with same bases and between the same parallels have equal bases.



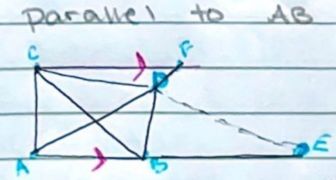
Area(ABC)
= 1/2 Area(ABEC)
= 1/2 Area(ABFD)
= Area(ABD) //

I-38: Triangles with equal bases and between the same parallels have equal bases

I-39 Triangles with the same bases and with equal area are between the same parallels [ie. have the same height]



Given ΔABC & ΔABD, with equal areas, AB∥CD
Assume by way of contradiction, that CD is not parallel to AB



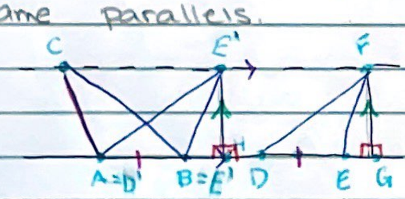
Draw a parallel to AB through C & let F be the point where BD meets the new line

Connect A to F to make $\triangle ABF$.
By I-37, $\triangle ABF$ & $\triangle ABC$ have equal areas

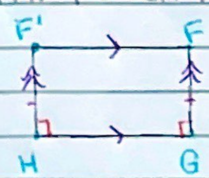
Case 1: If D is between B & F, $\triangle AED$ is part of $\triangle ABF$
Case 2: If F is between B & D, then $\triangle ABF$ is part of $\triangle ABD$

In either case, $\triangle ABD$ & $\triangle ABF$ have different areas and hence $\triangle ABD$ has area different from $\triangle ABC$, contradicting the assumption they are equal
∴ CD must be parallel to AB //

I-40: Triangles on equal bases (on the same side of the same line) and with equal areas are between the same parallels.



proof:
Apply $\triangle DEF$ to $\triangle ABC$ so that $BC = D'E'$ & then
$CF' \parallel AB \parallel DE$ by I-39
Why is F on CF' too?
By angle-angle-side congruence, $\triangle BHF' \cong \triangle EGF$
so $|HF'| = |GF|$



Feb. 6, 2024