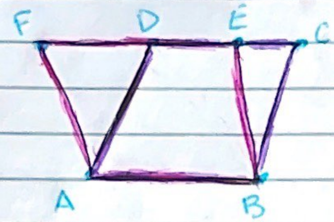
I-35 Two parallelograms with the same base and between the same parallels have equal areas.

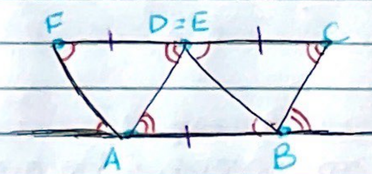


Proof:

we have $\square$s ABCD & ABEF with CDEF on the same line

We'll work by cases, based on how the $\square$s overlap.

↳ case 0: D = E



$|AB| = |FD| = |CD|$

By S-A-S $\triangle DAB \cong \triangle BCD \cong ADF$

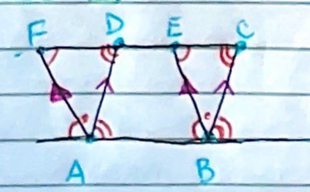
area(ABCD)

= area($\triangle ABD$ + $\triangle DCB$)

= area($\triangle ABD$ + $\triangle ADF$)

= area(ABEF) //

↳ case 1:



must have $|AF| = |BE|$ & $|AD| = |BC|$

$\angle \cdot = \angle$, $\triangle = \triangle$ & $|AD| = |BC|$

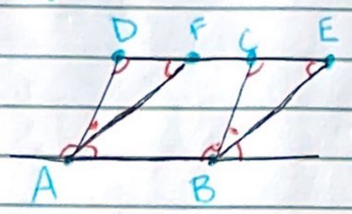
$\Rightarrow \triangle BCE \cong \triangle ADF$

Area (ABCD) = Area (ABED) + Area($\triangle BCE$)

= Area (ABED) + Area ($\triangle ADF$)

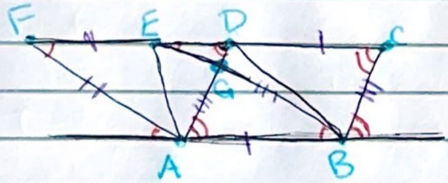
= Area (ABEF)

↳ Case 2:



S-A-S (using $|AF| = |BE|$ & $\angle = \angle$, & $\angle = \angle$)
& decompose similarly to case 1. //

↳ Case 3:



By S-A-S, $\triangle ADF \cong \triangle BCE$

$$|\square ABEF| = |\triangle ADF| + |\triangle ABG| - |\triangle GED|$$
$$= |\triangle BCE| + |\triangle ABG| - |\triangle GED|$$
$$= |\square ABCD|$$ //

Feb. 1st 2024