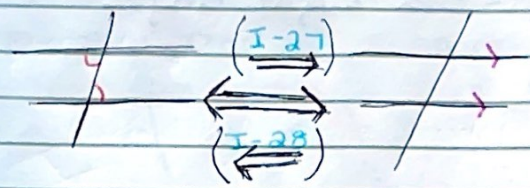
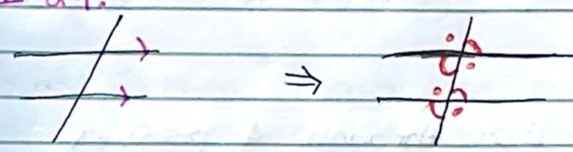
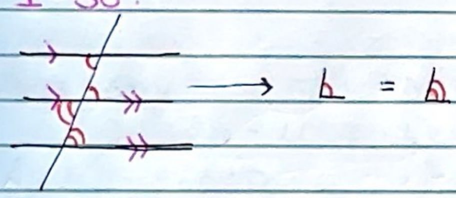
Recall: prop I-27 & 28 (ie the Z theorem)



I-29:

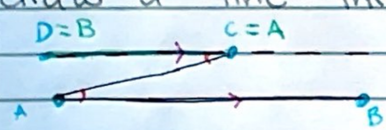


I-30:



If two lines are each parallel to a third, they are also parallel to each other

I-31: Given a line AB and a point C not on (any extension of) AB, we can draw a line through C parallel to AB.



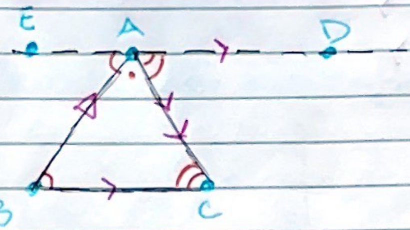
Proof:

Draw a line from A to C. Consider ∠CAB. Draw a copy of ∠CAB at C so that AC follows along CA and the rest of the new angle is on the other side of CA from B. this makes alternate interior angles, so AB ∥ AB

↳ parallel to.

February 1st 2024

I-32: The sum of the interior angles of any triangle is 2 right angles



Proof

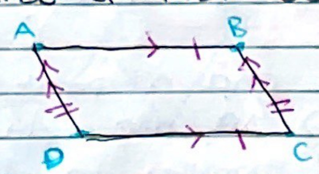
Draw a line ED through A parallel to BC. Then $\angle EAB = \angle ABC$ & $\angle DAC = \angle ACB$ by the Z theorem.

$\angle ABC + \angle ACB + \angle BAC$

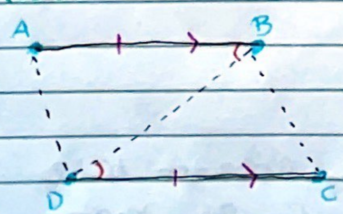
$= \angle EAB + \angle DAC + \angle BAC$

$= \angle EAD = 2 \perp$

I-33: Suppose $AB \parallel DC$ & $|AB| = |DC|$ then $AD \parallel BC$ & $|AD| = |BC|$



Proof:



Draw BD, then by the Z-theorem.

$\angle ABD = \angle CDB$.

Since $|AB| = |DC|$ and $|BD| = |DB|$ S-A-S gives us $\Delta ABD \cong \Delta CDB$

Thus $|AD| = |CB|$ & $\angle CBD = \angle ADB \Rightarrow$ by Z-theorem, $AD \parallel CB$

I-34: In any parallelogram, opposite sides are equal in length, and opposite angles are equal and a diagonal "cuts it in half"

Proof:

Given $\square ABCD$, Draw the diagonal BD.

By the Z-theorem, $\angle ABD = \angle CDB$ & also $\angle ADB = \angle CBD$. Thus $\angle ABC = \angle ABD + \angle CBD$

$$= \angle CDB + \angle ADB$$
$$= \angle ADC$$

& $\angle BAD = 2\llcorner - \angle ABD - \angle ADB$

$$= 2\llcorner - \angle CDB - \angle CBD$$
$$= \angle BCD$$

By A-S-A $\cong$ we have $\triangle ABD \cong \triangle CDB$ because $\angle ABD = \angle CDB$ & $|BD| = |DB|$ & $\angle ADB = \angle CBD$

So $|AD| = |CB|$ & $|AB| = |CD|$ //