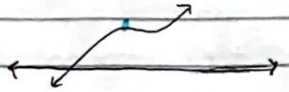
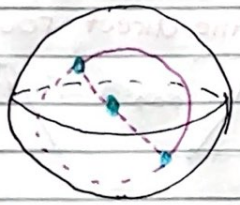
What happens if the parallel axiom fails? (a side for the main course...)

Failure mode 1:
There is more than one line through the point parallel to the given line

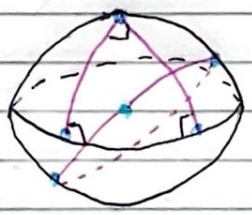
Failure mode 2:
There is no line through the point parallel to the given line.



A model of such an "elliptic" geometry.



"points" are opposite pairs of points on the sphere
"lines" are the great circles of the sphere



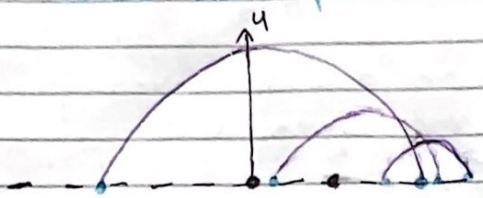
Sum of the internal angles of a triangle $> 2\pi$
Area of $\triangle ABC$ = (Sum of the internal angles $- \pi$)

Jan. 25, 2024

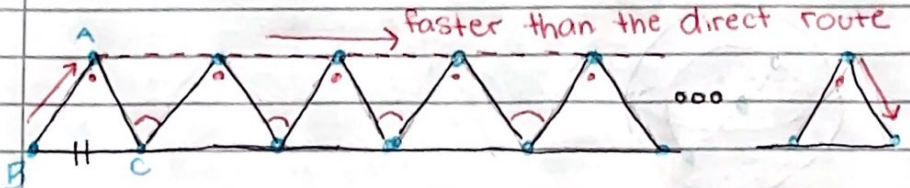
Lecture 9: Poin Caré half-plane model

Failure mode 1: Suppose that there is more than one parallel line.
"Hyperbolic geometry"

Poin Caré half-plane model



"points" are the points of $\mathbb{R}^2$ with $y>0$
"lines" are upper-semi circles with centres on the x-axis
Hence the sum of the internal angles of a triangle is always less than 2π.
Area of $\triangle ABC$: π - (sum of the internal angles)
Why Postulate II must be modified if there are no parallel lines:



$\Rightarrow \angle < \angle \Rightarrow \ldots + \cdot 2 \rightarrow \nparallel$