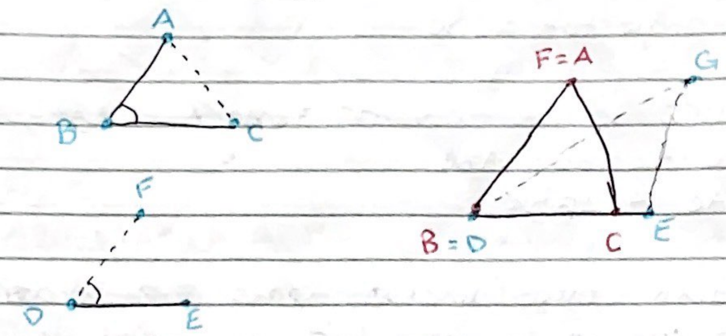
Lecture 8: Proposition I-23, I-24 and I-25

Jan. 25, 2024

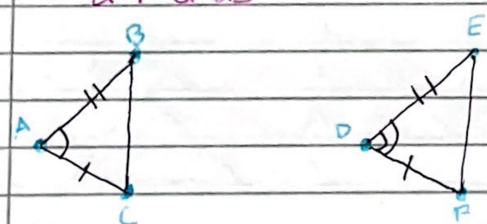
I-23: Given a rectilinear angle $\angle ABC$ and a line segment DE construct F Such that $\angle FDE = \angle ABC$



Proof

Connect A to C to make $\triangle ABC$
Pick a point G not on any extension of DE. Connect G to D & E to make $\triangle DEG$.
Apply $\triangle ABC$ to $\triangle DEG$ So that B is on D and BC lies along DE and A is on the same side of DE as G.
Then let F = A and obviously
$\angle FDE = \angle ABC$ //

I-24 & 25

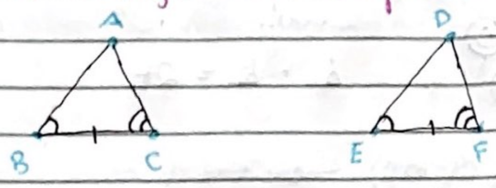


Given: $|AB| = |DE|$ & $|AC| = |DF|$
Then: $\angle EDF < \angle BAC \iff |EF| < |BC|$

$$|EF|^2 = |DE|^2 + |DF|^2 - 2|DE||DF|\cos(\measuredangle)$$

Page 18

I-26 (Angle-side-angle) congruence criterion)



If $|BC| = |EF|$, and $\angle ABC = \angle DEF$, and $\angle ACB = \angle DFE$ then $\triangle ABC \cong \triangle DEF$

Proof:

apply $\triangle ABC$ to $\triangle DEF$ so that B is on E & BC is along EF & A is on the same side of EF as D. Since $|BC| = |EF|$, C is on F.

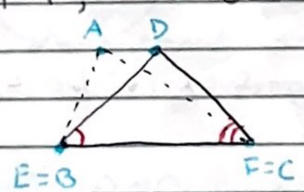
Since $\angle DEF = \angle ABC$ & BC coincides with EF & A & D are on the same side of EF we must have BA lie along ED.

Similarly, CA lies along FD it follows that $A = BA \cap CA$

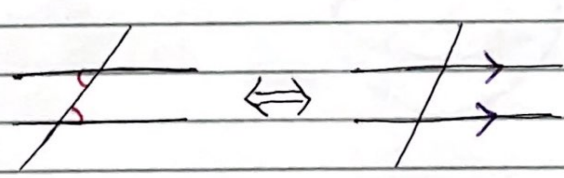
$= ED \cap FD$

$= D$

so A is on D... B is on E and C is on F, $\triangle ABC \cong DEF$



I-27 & 28



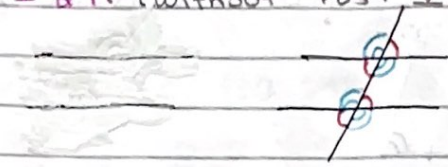
A line falling across two other lines make equal alternate angle <u>if</u> the two other lines are parallel

$\Rightarrow$ (I-27) does not need Post. V

Lecture 8 Proposition I-27 & I-28 Continued

Jan. 25, 2024

I-27. (without Post V)

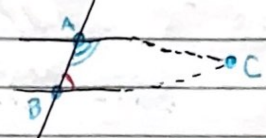


$\angle + \angle = 2\llcorner$

If the angles are equal, then the lines do not intersect

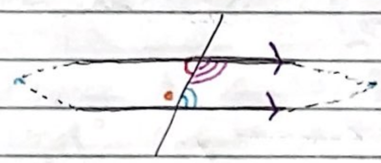
Proof

assume by way of contradiction, that the lines intersect.



$\Rightarrow \angle + \angle < 2\llcorner$ ⊗

I-28. (with Post. V)



To Show:

$\angle = \angle$

Proof:

We know by Post. V that if $\angle + \angle < 2\llcorner$, then the lines will cross on that side. Similarly, if $\angle + \angle < 2\llcorner$ they'd cross on that side.

If $\angle + \angle$ can't be $< 2\llcorner$ and can't be $> 2\llcorner$ then $\angle + \angle = 2\llcorner$ but $\angle + \angle = 2\llcorner$ $\therefore \angle = \angle$