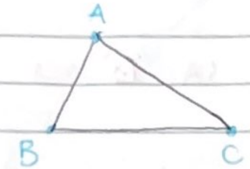
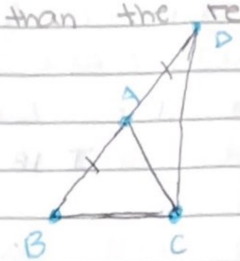
Proposition I-20 and I-21 : Lecture 7

Jan 23 2024

I-20: In any triangle, the sum of any two sides is greater than the remaining side.



or



$|AB| + |AC| > |BC|$ (use second diagram for proof)

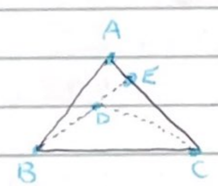
Proof:

1. Extend AB past A to D such that $|AD| = |AC|$
2. Connect C to D to make $\triangle DBC$ & $\triangle ACD$.

If we can show that $\angle DCB > \angle BDC$, then $|BC| < |BD| = |AB| + |AD| = |AB| + |AC|$

3. But $\angle DCB = \angle DCA + \angle ACB$
$= \angle CDA + \angle ACB$
$= \angle BDC + \angle ACB > \angle BDC$

I-21: Suppose the point D is inside $\triangle ABC$ if we connect D to B & C to make $\triangle DBC$, then $\angle BDC > \angle BAC$, $|AB| > |DB|$, and $|AC| > |DC|$.



Proof: (side lengths)

1. Extend BD past D to E on AC.

Then $|BE| < |BC| + |CE|$
$< |AB| + |AE|$ (use this one).

So $|BE| + |EC| < |AB| + |AE| + |EC|$

$|BE| + |EC| < |AB| + |AC|$

$|BD| + |DE| + |EC| < |AB| + |AC|$

$\vee$

$|BD| + |DC| < |AB| + |AC|$

it remains to show that $\angle BAC < \angle BDC$

$\angle BDC < \angle BDC$.

$\angle BDC > \angle DEC = \angle BEC$

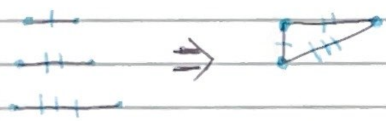
external angle for $\triangle EDC$

But $\angle DEC$ is an external angle for $\triangle ABE$,

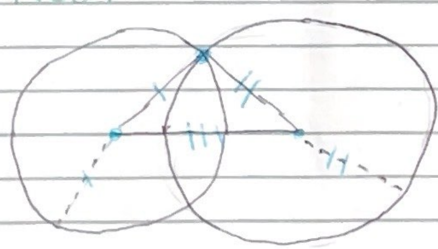
So $\angle DEC > \angle ABE = \angle ABC$

$\therefore \angle BAC < \angle BDC$

I-22: Given three line segments such that any two add up to more than the third, we can use these line segments as the sides of the same triangle.



Proof:



Proposition I-21 continued and I-22: Lecture 7

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