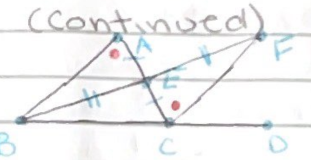
I-16 (continued)



To Show: $\angle ABC < \angle ACD$ & $\angle BAC < \angle ACD$ too.

Proof

1. Let E be the midpoint of AC. Connect B to E & extend to F such that $|BE| = |FE|$
2. Connect F to C. Consider ΔEFC & ΔEBA.

These are congruent by S-A-S.

$|BE| = |FE|$ (by construction)

$|CE| = |AE|$ (midpoint)

$\angle AEB = \angle CEF$ (opposite angle theorem)

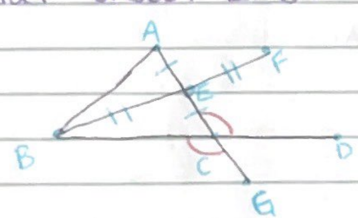
$\therefore \Delta EFC \cong \Delta EBA$

$\therefore \angle BAE = \angle FCE$

$$\begin{aligned}\angle ACD &= \angle ACF + \angle FCD \\ &= \angle FCE + \angle FCD \\ &= \angle BAE + \angle FCD \\ &= \angle BAC + \angle FCD\end{aligned}$$

$\therefore \angle ACD > \angle BAC$

What about $\angle ABC < \angle ACD$?



Extend AC past C to G

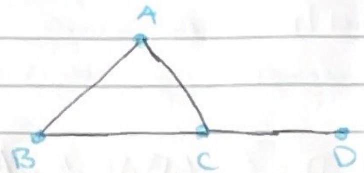
By opposite angle theorem $\angle ACD = \angle BCG$

But $\angle ABC$ is to $\angle BCG$ as $\angle BAC$ was to $\angle ACD$ by the same argument ("Mutatis mutandis")

$\angle ABC < \angle BCG = \angle ACD$.

Lecture 6: Proposition I-16 – continued

I-17: To Show: $\angle ABC + \angle BCA < 2\llcorner = \frown$



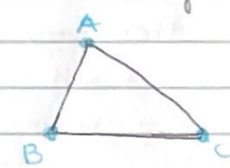
Proof:

$\angle BCA + \angle ACD = 2\llcorner = \frown$ Since BCD are on the same straight line.

$\angle ABC + \angle BCA < \angle ACD + \angle BCA$ (by I-16)

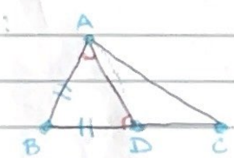
$$\angle ACD + \angle BCA = 2\llcorner$$

I-18: In any triangle the greater side subtends the greater angle



$|AB| < |BC| \Rightarrow \angle ACB < \angle BAC$

Proof:



$|BD| = |AB|$ D is between B & C

$\angle BAD < \angle BAC$

Isosceles → || → external $\angle$ of $\triangle ACD$

$\angle BDA > \angle ACD$

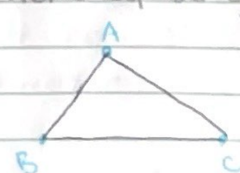
$\qquad\qquad ||$

$\angle BDA > \angle ACB$ //

I-19: In any triangle the greater angle is subtended by the greater side $\angle ACB < \angle BAC$

$$\Downarrow$$

$|AB| < |BC|$



proof: (By contradiction)
Suppose $\angle ACB < \angle BAC$ but $|AB| \geq |BC|$
If $|AB| = |BC|$, ΔABC would be isosceles
and have $\angle BAC = \angle ACB$ ⊗
If $|AB| > |BC|$, by I-18, $\angle ACB > \angle BAC$ ⊗ //