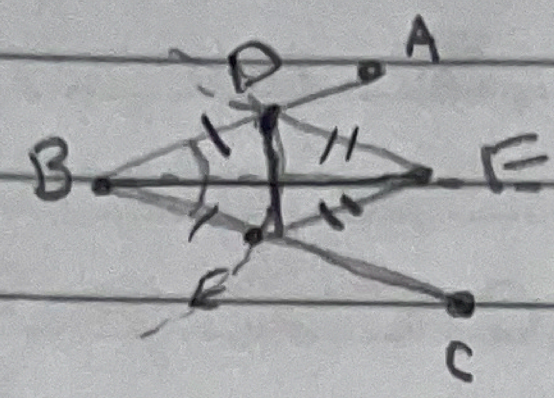
Lecture 4: Proposition I-9 & I-10

Jan. 16 2024

I-9: Given an angle, cut it in half



So bisecting an angle can be done with compass and straightedge trisecting usually can't

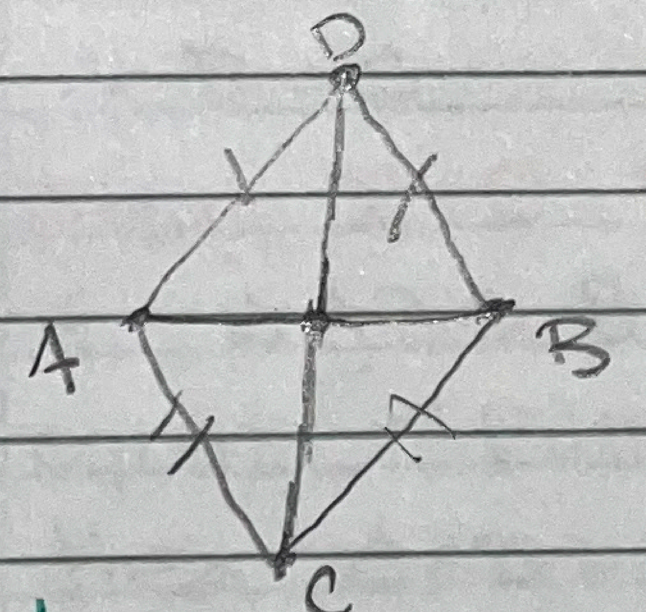
Proof:

1. draw a circle with centre B intersecting AB and AC at D & E respectfully
2. Construct an equilateral triangle $\triangle DEF$
3. Connect B and F
4. $|BD| = |BE|$ (radii) & $|DF| = |EF|$ (sides of equilateral triangle & $|BF| = |BF| \Rightarrow \triangle BDF \cong \triangle BEF$ (by S-S-S I-8)

$\therefore \angle ABF = \angle BDF = \angle EBF = \angle CBF$

I-10: Given a line segment, cut it in half

Proof: Given AB Find C between A & B so that $|AC| = |BC|$



1. construct equilateral triangle with base AB on both sides of AB say $\triangle ADB$ & $\triangle AEB$
2. Connect D & E and let C be the intersection of AB & DE

Claim: $|AC| = |BC|$

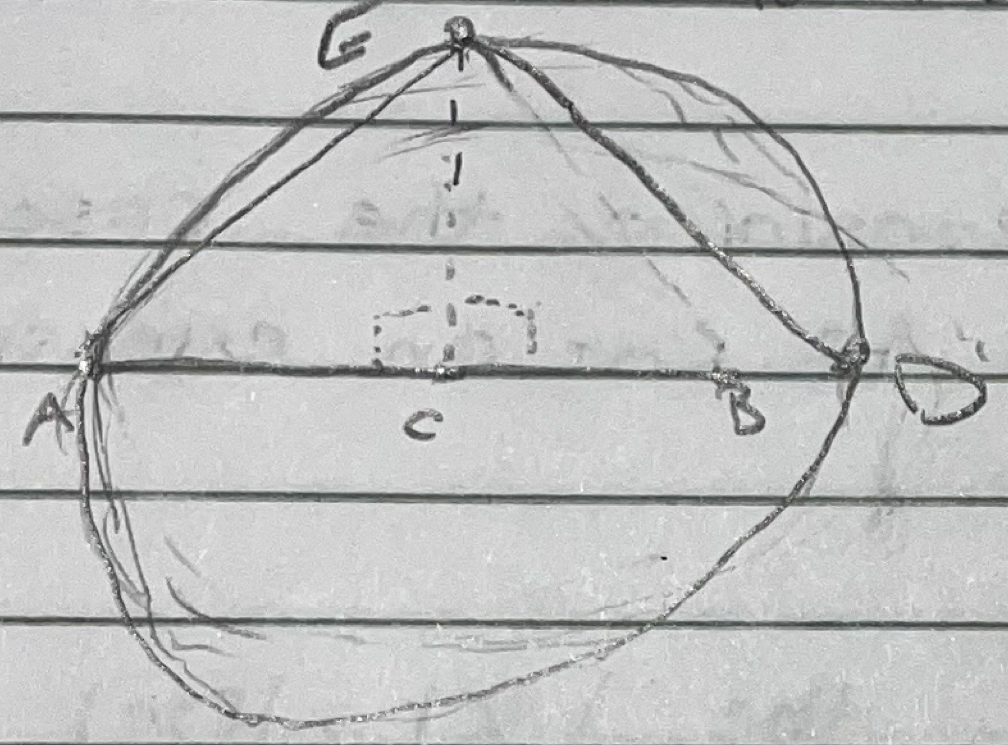
First $\triangle DAE \cong \triangle DBE$ by S-S-S because $|DA| = |DB|$, $|AE| = |BE|$, $|DE| = |DE|$

Second: $\triangle DAE \cong \triangle DBE$
$\Rightarrow \angle ADE = \angle BDE$
$\quad\;\; \shortparallel \qquad\qquad \shortparallel$
$\angle ADC \qquad \angle BDC$

Third: $|AD| = |BD|$ & $|DC| = |DC|$ & $\angle ADC = \angle BDC$ so $\triangle ADC \cong \triangle BDC$ by S-A-S (prop I-4)

Fourth $\triangle ADC \cong \triangle BDC$
$\Rightarrow |AC| = |BC|$ //

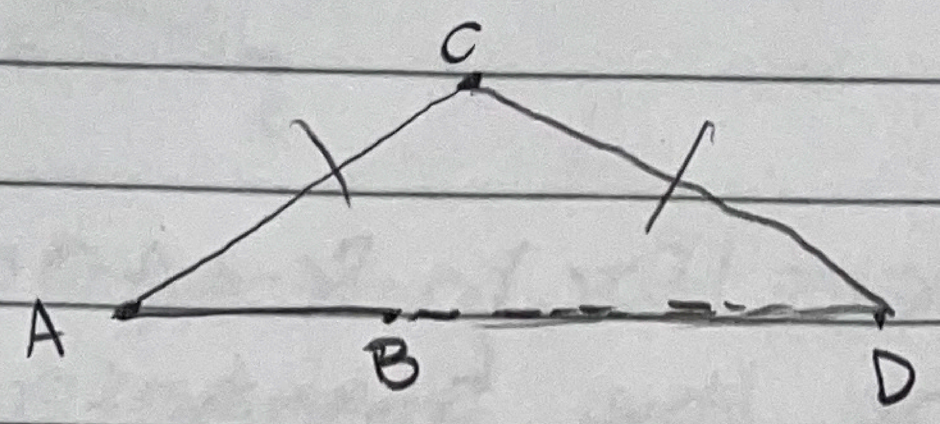
I-11: At a given point on a line, construct a line perpendicular to the given one.



Proof:
1. Draw a circle with radius AC & centre C (extend AB past B until it meets the circle) meeting CB at D.
2. Construct an equilateral triangle $\triangle ADE$ on AD. Connect E to C.
3. $\triangle AEC \cong \triangle DEC$ since $|AE| = |DE|$ (sides of an equilateral $\triangle$) & $|EC| = |EC|$ & $|CA| = |CD|$ (radii)

$\therefore \angle ACE = \angle DCE$ but $\angle ACE + \angle DCE = \angle ACD$ = straight angle
$\therefore \angle ACE = \angle DCE$ are right angles by definition.

I-12: Given a line AB and a point C (not any extension of AB) Construct a line through C perpendicular to AB.



Proof:

1. Join C to A, and draw a radius with circle with radius CA & Centre C. If the circle only touches (any extension of) AB at A, then pick a different point of AB to draw a radius to.

2. So we need only consider the case where the circle intersects AB (or an extension) at two points, A & D.

3. connect C to D, the $|AC| = |DC|$ (radii)
4. Let E be the midpoint of AD, so $|AE| = |DE|$ Since $|CE| = |CE|$ $\triangle ACE \cong \triangle DCE$, so $\angle CEA = \angle CED = \llcorner$