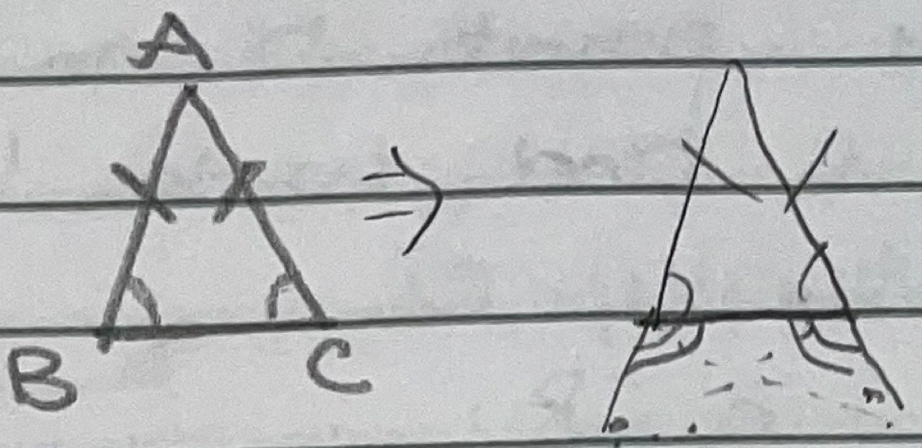
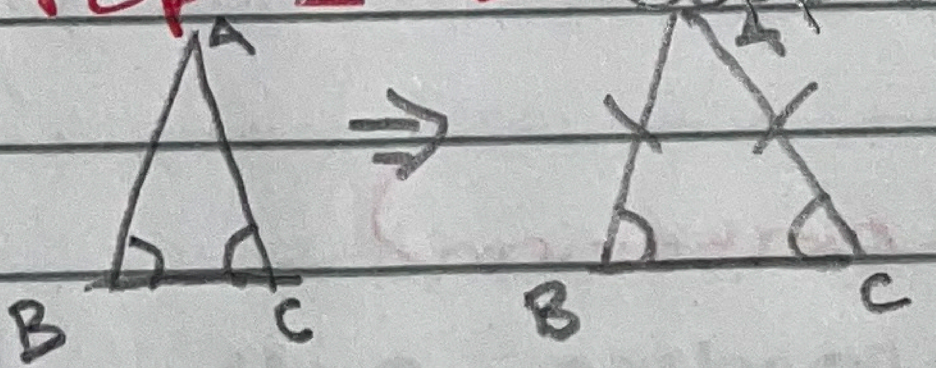
Since the relocated $\triangle ABC$ is vertex-for-vertex $\triangle DEF$ all corresponding angles and side lengths are equal

Prop I-5: The base angles of an isosceles triangle are equal

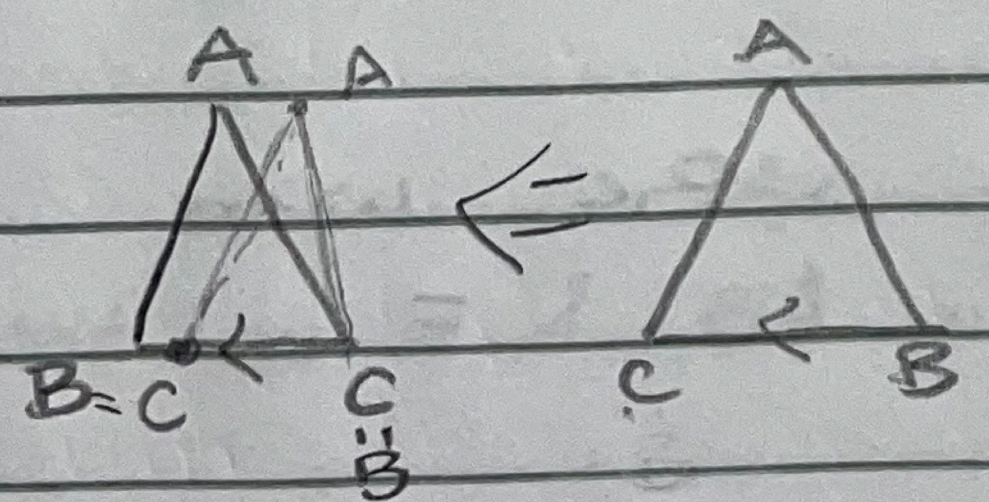



Proof:
$|AB| = |AC|$ is given
[To show: $\angle ABC = \angle ACB$]
Consider $\triangle ABC$ & $\triangle ACB$
we have $|AB| = |AC|$ (given)
& $|AC| = |AB|$ (-"-)
& $\angle BAC = \angle CAB$
By side-angle-side congruence (I-4)
we have $\triangle ABC \cong ACB$
$\therefore$ Corresponding angles are equal for the two triangles
In particular, $\angle ABC = \angle ACB$ //

Prop I-6: Suppose $\triangle ABC$ has $\angle ABC = \angle ACB$ then $|AB| = |AC|$



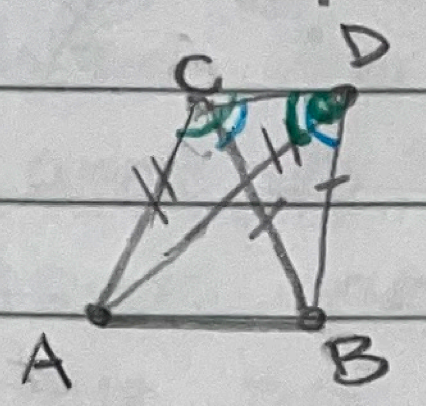

Proof: Apply $\triangle ACB$ to $\triangle ABC$




with B being put on C & BC lying along CB & A on the same side of CB as A
Since $|BC| = |CB|$ & $|BC|$ lies along CB & B is on C, we must have C on B.
Since $\angle ACB = \angle ABC$ CA lies on BA & since $\angle ABC = \angle ACB$, BA lies on CA
$\therefore A = CA \cap BA = BA \cap CA = A$

Since A is on A, B is on C, and C is on B, we have $|AB| = |AC|$

**Prop I-7:** Suppose we are given AB and C (which is not any extension of AB). Then any point D on the same side of AB as C and with $|AD| = |AC|$ & $|BD| = |BC|$, is actually C.




Suppose by way of contradiction that $C \neq D$

Since $|AC| = |AD|$ & $|BC| = |BD|$, $\triangle ACD$ & $\triangle BCD$ are isosceles

It follows by I-6, that $\angle ACD = \angle ADC$ and $\angle BCD = \angle BDC$

Then in the picture, $\angle BCD < \angle ACD$ and $\angle ADC < \angle BDC$

So $\angle BCD < \angle BDC$ & $\angle BCD = \angle BDC$ which is absurd.

$\therefore D = C$

**Prop I-8:** (S-S-S congruence criterion)

if $\triangle ABC$ & $\triangle DEF$ have corresponding sides equal $[|AB| = |DE|, |AC| = |DF| \text{ \& } |BC| = |EF|]$, then $\triangle ABC \cong \triangle DEF$

**Proof:** Apply $\triangle DEF$ onto $\triangle ABC$ with D on A, and DE lies along AB, & F on the same side of AB as C