Mathematics 2260H – Geometry I: Euclidean Geometry

TRENT UNIVERSITY, Winter 2023

Assignment #5 The Pythagorean Theorem Due on Friday, 17 February.*

The Pythagorean Theorem states that if one constructs squares on the sides of a right-angled triangle, then the area of the square on the hypotenuse is equal to the sum of the areas of the squares on the other two sides. Equivalently, in modern terms, if a and b are the lengths of the short sides of a right triangle and c is the length of the hypotenuse, then $a^2 + b^2 = c^2$. The Pythagorean Theorem has hundreds of different proofs, Euclid's being one of the less transparent ones (see Proposition I-47).

1. Find, by looking it up or by doing it yourself, and present the simplest – for full credit, both conceptually and in terms of prior knowledge it requires – proof of the Pythagorean Theorem that you can. [5]

Much less well known is the converse of the Pythagorean Theorem, that if one constructs squares on the sides of a triangle and two of them have their areas sum to the area of the third, then the triangle is a right triangle. Equivalently, if the sides of a triangle have lengths a, b, and c, and $a^2 + b^2 = c^2$, then the triangle is a right triangle. You can take alook at Euclid's proof by looking at Proposition I-48, the last one in Book I of the *Elements*.

2. Find, by looking it up or by doing it yourself, and present the simplest – for full credit, both conceptually and in terms of prior knowledge it requires – proof of the converse of the Pythagorean Theorem that you can. [5]

^{*} If submitting on paper or on Blackboard isn't feasible, please email your solutions to the instructor at: sbilaniuk@trentu.ca