Mathematics 2260H – Geometry I: Euclidean Geometry
TRENT UNIVERSITY, Winter 2023

Assignment #2
Congruence and Similarity
Due on Friday, 27 January.[*]

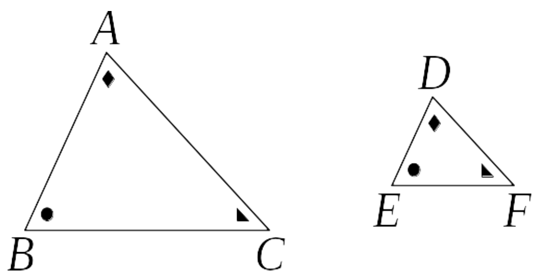
The Angle-Side-Side congruence criterion asserts that if we are give triangles $\triangle ABC$ and $\triangle DEF$ with $\angle ABC = \angle DEF$, $|BC| = |EF|$, and $|CA| = |FD|$, then $\triangle ABC \cong \triangle DEF$.

Recall that an angle is *acute* if it is less than a right angle and *obtuse* if it is greater than a right angle.

1. Give an example to show that the Angle-Side-Side congruence criterion may fail when the angle is acute. *[2]*

2. Explain (informally!) why the Angle-Side-Side congruence criterion works if the angle is a right or obtuse angle. *[1]*

A close relative of congruence is the notion of similarity[†]. Two triangles, say $\triangle ABC$ and $\triangle DEF$, are said to be *similar*, written as $\triangle ABC \sim \triangle DEF$, if corresponding angles are equal, *i.e.* if $\angle ABC = \angle DEF$, $\angle BCA = \angle EFD$, and $\angle CAB = \angle FDE$. Intuitively, this means that the triangles have the same shape, but may be of different sizes; scaling one up or down can make it congruent to the other.



Since we haven't gotten through enough of Euclid's *Elements* to handle similarity his way, we will develop it using trigonometry instead. You may also assume that the sum of the interior angles of a triangle is equal to two right angles. (This turns out to be equivalent to Postulate V, by the way.) In particular, you may assume the Law of Sines for triangles, *i.e.* given any $\triangle ABC$,

$$\frac{\sin(\angle BAC)}{|BC|} = \frac{\sin(\angle ABC)}{|AC|} = \frac{\sin(\angle ACB)}{|AB|},$$

as well as the Cosine Law for triangles, *i.e.* given any $\triangle ABC$,

$$|BC|^2 = |AB|^2 + |AC|^2 - 2 \cdot |AB| \cdot |AC| \cdot \cos(\angle BAC).$$

[*] If submitting on paper or on Blackboard isn't feasible, please email your solutions to the instructor at: `sbilaniuk@trentu.ca`

[†] So closely are they related that congruence and similarity are the same thing when Postulate V fails!

3. (Angle-Angle Similarity Criterion) Show that if $\angle ABC = \angle DEF$ and $\angle BCA = \angle EFD$ in triangles $\triangle ABC$ and $\triangle DEF$, then $\triangle ABC \sim \triangle DEF$. *[1]*

4. Show that if $\triangle ABC \sim \triangle DEF$, then $\frac{|AB|}{|DE|} = \frac{|BC|}{|EF|} = \frac{|AC|}{|DF|}$. *[2]*

4. (Side-Side-Side Similarity Criterion) Show that if $\frac{|AB|}{|DE|} = \frac{|BC|}{|EF|} = \frac{|AC|}{|DF|}$ in triangles $\triangle ABC$ and $\triangle DEF$, then $\triangle ABC \sim \triangle DEF$. *[2]*

5. (Side-Angle-Side Similarity Criterion) Show that if $\angle BAC = \angle EDF$ and $\frac{|AB|}{|DE|} = \frac{|AC|}{|DF|}$ in triangles $\triangle ABC$ and $\triangle DEF$, then $\triangle ABC \sim \triangle DEF$. *[2]*