Mathematics 2260H – Geometry I: Euclidean Geometry
Trent University, Winter 2023

Assignment #10 – Menelaus' and Ceva's Theorems
Due on Friday, 24 March.

Recall that a *cevian* of a triangle is a line joining a vertex to a point on (an extension of) the opposite side. We proved the following in class on 2023-03-21:

Ceva's Theorem. If AD, BE, and CF are cevians of $\triangle ABC$, then they are concurrent in a point S if and only if $\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = +1$.

The components of the product in the the theorem use the convention that if points X, Y, and Z are on the same line, then

$$\frac{XY}{YZ} = \begin{cases} +\frac{|XY|}{|YZ|} & \text{if } Y \text{ is between } X \text{ and } Z \\ -\frac{|XY|}{|YZ|} & \text{otherwise} \end{cases} .$$

1. Use Ceva's Theorem to show that the medians of a triangle are concurrent. *[2]*

On the same day, we also proved the following, using the same convention about ratios of line segments:

Menelaus' Theorem. Suppose P, Q, and R are points on (extensions of) sides AB, BC, and AC of $\triangle ABC$. Then P, Q, and R are collinear if and only if $\frac{AP}{PB} \cdot \frac{BQ}{QC} \cdot \frac{CR}{RA} = -1$.

2. Use Menalaus' Theorem to prove the forward direction of Ceva's Theorem, *i.e.* that if AD, BE, and CF are cevians of $\triangle ABC$ that are concurrent in a point S, then $\frac{AF}{FB} \cdot \frac{BD}{DC} \cdot \frac{CE}{EA} = +1$. *[8]*

Hint: You will need to apply Menelaus' Theorem to some of the subtriangles the cevians meeting at S divide $\triangle ABC$ into.