Euclid's Postulates Extended

For convenience, here are Euclid's five Postulates from Book I of the *Elements*, plus two more Postulates that fill two of the most obvious gaps that occur in Book I. (The extra two have been adapted from §2.2 of *Geometry from Euclid to Knots*, by Saul Stahl.) In what follows below, recall that to Euclid "line" means what we mean by "curve" and that all "lines" are of finite length (though they might be extensible, as in, for example, Postulate II).

The Postulates

I. One can draw a straight line from any point to any point.

II. One can extend any straight line continuously in a straight line as far as one likes in either direction.

III. One can draw a circle with any centre and radius.

IV. All right angles are equal to one another.

V. If a straight line falling across two other straight lines makes the sum of the internal angles on the same side less than the sum of two right angles, the two straight lines, if extended indefinitely, intersect on that side of the original straight line that the sum of the internal angles is less than the sum of two right angles.

We will occasionally use the following equivalent of Postulate V, sometimes called Playfair's Postulate:

V′. Given an infinite straight line and a point not on that line, there is exactly one infinite straight line through that point that goes not intersect the given straight line.

Here are the two additional Postulates, the Separation and Application Postulates, respectively:

S. Any infinite straight line, any circle, and any triangle separate the plane into two regions such that any line joining a point in one region to a point in the other region intersect the separating line.

Note that a triangle or circle, or indeed any curve, could be a "line joining a point in one region to a point in the other region".

A. Given two triangles $\triangle ABC$ and $\triangle DEF$, it is possible to apply $\triangle ABC$ to $\triangle DEF$ (*i.e.* place a copy of $\triangle ABC$ on $\triangle DEF$) so that vertex A falls on vertex D, side AB falls on side DE, and vertex C falls on the same side of DE as F does.

Postulates S and A can both be generalized quite a bit, at the cost of having more complex statements and/or more complex definitions behind them. The given versions should suffice for most elementary purposes. Please note that even with these additions, the system of postulates is not enough to be completely rigorous, but the extra two do fill the most obvious gaps that turn up in Book I of the *Elements*. For a properly complete set of postulates for Euclidean geometry, see the handout *Hilbert's Axioms for Euclidean Geometry*.