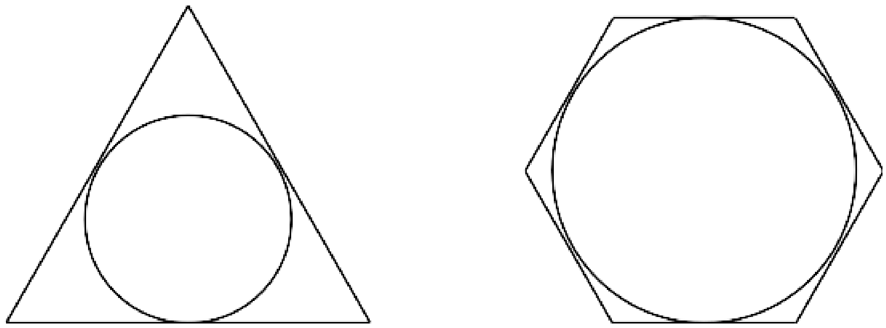
Mathematics 2260H – Geometry I: Euclidean Geometry
Trent University, Winter 2021

Solution to Assignment #2*e*
Inscribed Circles

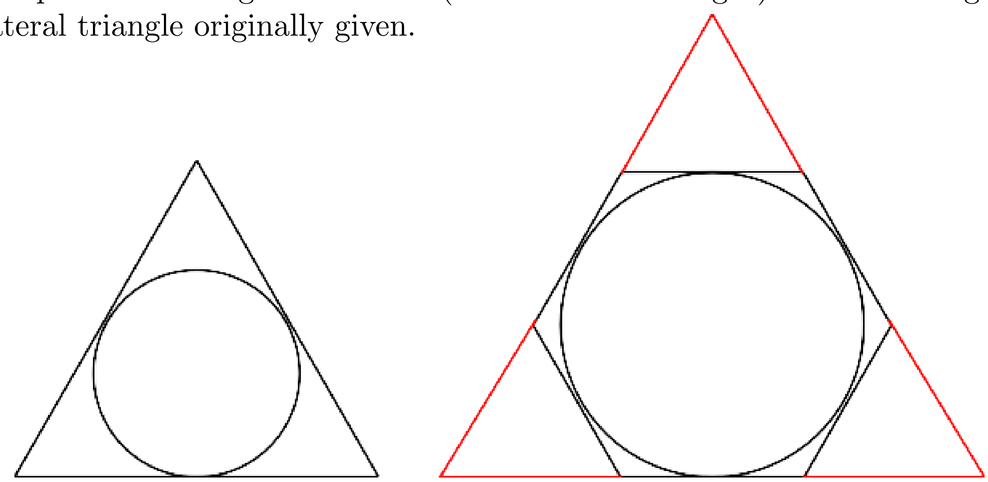
Due on Friday, 26 February.

Given twelve line segments of equal length, six are used to make an equilateral triangle and six are used to make a regular hexagon. Circles are inscribed inside each polygon, touching each side of its respective polygon at the midpoint only.



1. Find the ratio of the area of the circle inscribed in the triangle to the area of the circle inscribed in the hexagon. *[10]*

Solution. Observe that the equilateral triangle can be decomposed into four smaller equilateral triangles with side lengths equal to the lengths of the original twelve line segments, while the regular hexagon can be decomposed into six such triangles. Consider the following diagram, in which three more such small triangles are added to the hexagon to turn into an equilateral triangle with sides (and width and height) 1.5 times longer that of the equilateral triangle originally given.



Since the (similar!) triangles they are contained in are scaled in a $2:3$ ratio, the circle's radii are scaled in the same ration. It follows that the larger circle, originally inscribed in the hexagon, has $\left(\frac{3}{2}\right)^2 = \frac{9}{4} = 2.25$ the area of the smaller one; equivalently, the smaller one has $\left(\frac{2}{3}\right)^2 = \frac{4}{9} = 0.\dot{4} = 0.4444\ldots$ the area of the larger one. ■

This is an extra assignment which, should you do it, will expand the pool from which your best ten assignments are chosen. Enjoy your Reading Week!