Mathematics 2260H – Geometry I: Euclidean Geometry
Trent University, Winter 2021

Assignment #3 – A centre for a regular n-gon
Due on Friday, 5 February.

Recall that a regular polygon is one with all sides of equal length and all internal angles equal. A polygon with n sides is often referred to as an *n-gon*.* In what follows, suppose $A_1A_2 \dots A_n$ is a regular n-gon in the Euclidean plane for some $n \geq 3$.

1. Let ℓ_1, ℓ_2, ... , and ℓ_n be the lines bisecting (*i.e.* cutting in half) the interior angles at A_1, A_2, ... , and A_n, respectively, of the regular n-gon $A_1A_2 \dots A_n$. Show that ℓ_1, ℓ_2, ... , and ℓ_n are *concurrent*, that is meet at a common point O. *[4]*

2. Let m_1, m_2, ... , and m_n be the perpendicular bisectors (*i.e.* lines cutting in half at a right angle) of the sides A_1A_2, A_2A_3, ... , and A_nA_1, respectively, of the regular n-gon $A_1A_2 \dots A_n$. Show that m_1, m_2, ... , and m_n are concurrent also concurrent at the point O in question **1**. *[4]*

3. Besides the regular polygon $A_1A_2 \dots A_n$, what else is the point O a centre of? *[2]*

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* For small n we have common names: triangle, quadrilateral, pentagon, and so on. Note that in the Euclidean and hyperbolic planes an n-gon with positive area must have $n \geq 2$, but in the elliptic plane there are 2-gons ("biangles"?) with positive area.