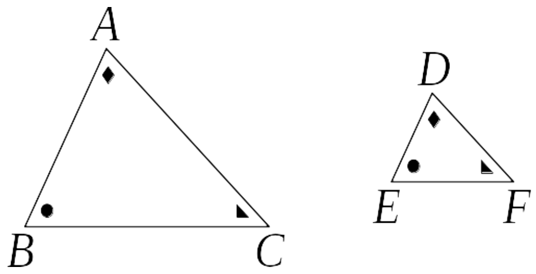
Mathematics 2260H – Geometry I: Euclidean Geometry
TRENT UNIVERSITY, Winter 2021

Assignment #2 – Similarity
Due on Friday, 29 January.

Two triangles are said to be *similar* if they have the same proportions, though not necessarily the same size. More formally, $\triangle ABC$ is similar to $\triangle DEF$, often written as $\triangle ABC \sim \triangle DEF$, if corresponding angles of the triangles are equal, *i.e.* $\angle ABC = \angle DEF$, $\angle BCA = \angle EFD$, and $\angle CAB = \angle FDE$.



Similarity is a very useful tool that we will develop a little earlier than Euclid does in the *Elements* by using trigonometry. You may – and definitely should! – assume thoughout this assignment that the sum of the interior angles of a triangle is equal to two right angles. (Or a straight angle, or π radians, or $180°$, or 200 gradians, or ... :-)

1. Prove the sine law for triangles, *i.e.* given any $\triangle ABC$,

$$\frac{\sin(\angle BAC)}{|BC|} = \frac{\sin(\angle ABC)}{|AC|} = \frac{\sin(\angle ACB)}{|AB|},$$

and the cosine law for triangles, *i.e.* given any $\triangle ABC$,

$$|BC|^2 = |AB|^2 + |AC|^2 - 2 \cdot |AB| \cdot |AC| \cdot \cos(\angle BAC) . \qquad \textit{[3]}$$

2. Show that if $\triangle ABC \sim \triangle DEF$, then $\dfrac{|AB|}{|DE|} = \dfrac{|BC|}{|EF|} = \dfrac{|AC|}{|DF|}$. *[2]*

Note that it follows from **2** that if two triangles are similar and have one pair of corresponding sides equal, then the triangles are congruent. (Why?)

3. Show that if $\dfrac{|AB|}{|DE|} = \dfrac{|BC|}{|EF|} = \dfrac{|AC|}{|DF|}$, then $\triangle ABC \sim \triangle DEF$. *[2]*

4. Show that if $\angle BAC = \angle EDF$ and $\dfrac{|AB|}{|DE|} = \dfrac{|AC|}{|DF|}$, then $\triangle ABC \sim \triangle DEF$. *[2]*

3 is the Side-Side-Side similarity criterion and **4** is the Side-Angle-Side similarity criterion for triangles. **5** is a congruence problem in which similarity may be useful:

5. Determine whether the Angle-Angle-Side congruence criterion for triangles always works. That is, given that $\angle CAB = \angle FDE$, $\angle ABC = \angle DEF$, and $|BC| = |EF|$, does it necessarily follow that $\triangle ABC \cong \triangle DEF$? *[1]*